Sacs.1101   Engineering Mathematics - I (2+1)   Marks:50

Time: 2 hours

I   **Fill in the blanks:**   (10x1=10)

1. If $z = \sin(2x + 3y^2)$ find $\frac{dz}{dx}$

2. Rodrigue's formula for $P_n(x)$ is _____.

3. $J_{1/2}(x) =$ _____.

4. Complementary function of $(D_2 - 4D + 3)y = 0$ is _____.

5. The total derivative of the function $z = f(x,y)$ is _____.

6. $\int_1^2 \int_0^1 4xy\, dx\, dy$ is _____.

7. Necessary and sufficient condition for the differential equation $Mdx + Ndy = 0$ to be exact is _____.

8. A vector with zero divergence is called _____.

9. For any vector function F, div curl F = _____.

       State True or False

10. The function $f(x,y) = \frac{xy^2 + x^3 - y^3}{yx^2 + xy^2}$ is a homogeneous function.

II   **Write Short notes on ANY FIVE of the following**   (5x2=10)

1. Expand $e^{\sin x}$ in ascending powers of x.

2. Verify Euler's theorem if $u = x^3 + y^3 - 3axy^2$.

3. Solve $(x+y-2)\, dx + (x - y + 4)\, dy = 0$.

4. Express $f(x) = 4x^3 - 2x^2 + 3x - 8$ in terms of Legendre polynomial.

5. Solve $(D^2 - 9)y = \sin 2x$.

6. Find Curl f, if $f = xz^3\, \vec{i} - 2x^2yz\, \vec{j} + 2yz^4\, \vec{k}$ at $(1,2,1)$

7. If a and b are irrotational, prove that $a \times b$ is irrotational.

III   **Answer ANY FIVE of the following**   (5x4=20)

1. Find $J\left(\frac{u,v,w}{x,y,z}\right)$ if $u = \frac{2yz}{x}$, $v = \frac{3zx}{y}$ and $w = \frac{4xy}{z}$

2. Find the maximum and minimum value of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

3. Solve $x^2y\, dx - (x^3 + y^3)\, dy = 0$

4. Solve $x\frac{dy}{dx} + y = x^3y^6$.

5. Prove that $J_{-n}(x) = (-1)^n J_n(x)$

6. Solve $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = \sin(\log x)$

7. Use Green's theorem to evaluate $\oint(x^2 + xy)dx + (y^2 + y^2)dy$ where C is the square formed by the lines $x = \pm 1$ and $y = \pm 1$.

IV   **Write an essay on ANY ONE of the following**   (1x10=10)

1. Verify Stoke's theorem for $f = (x^2 + y^2)\, \vec{i} - 2xy\, \vec{j}$ around the rectangle bounded by $x = \pm a$, $y = 0$ and $y = b$.

2. Solve by method of variation of parameters $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

****