KERALA AGRICULTURAL UNIVERSITY
B.Tech. (Food Engineering) - 2011 Admission 1st Semester
Final examination – February – March 2012

Cat. No: Basc 1102 | **Marks: 80**
Title: Engineering Mathematics -1 | **Time : 3 hours**

Part I

(Answer all questions)

1. State Cayley-Hamilton theorem.
2. Find the rank of the matrix $A = \begin{bmatrix} 4 & 9 & 3 \\ -1 & 0 & 2 \end{bmatrix}$.
3. If A is a square matrix, then $A - A^T$ is a _____ matrix.
   (a). unit (b). symmetric (c). skew-symmetric (d). non-singular
4. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, then the eigen values of A^{-1} are ____ and ____.
5. $\dfrac{\sqrt{x} - \sqrt{y}}{\sqrt{x} + \sqrt{y}}$ is a homogeneous function of degree _____.
   (a). 0 (b). 1 (c). 2 (d). 1.414
6. State Euler's theorem on homogeneous functions.
7. $\Gamma n =$ _____, if n is an integer.
8. The nature of the canonical form $2x^2 + 3y^2 - z^2 + 2xy + 4xz - 2yz$ is _____.
9. The sum of Eigen values of a matrix A is equal to _____.
   (a). Trace of A (b). $|A|$ (c). 1 (d). 0
10. Equation of the asymptotes of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is _____.

(10 x 1 = 10)

Part II

(Answer any ten questions)

11. Express the matrix $A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.
12. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

13. Expand e^x using Maclaurin's series containing terms up to x^4.

14. Evaluate $Lt_{x\to 0}\dfrac{\log x}{\cot x}$.

15. Find $\dfrac{\partial u}{\partial x}$ if $u=\tan^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$.

16. If $u=x^2-2y$ and $v=x+y$, find the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$.

17. Find the asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$.

18. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ by changing to polar co-ordinates.

19. Show that the vectors $(1,0,0)$, $(3,2,1)$ and $(1,-1,0)$ are linearly independent.

20. Obtain the diagonalised matrix associated with $A=\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$.

21. Compute $\Gamma\left(\frac{7}{2}\right)$.

22. Test for consistency: $2x+3y+5z=9$, $7x+3y-2z=8$ and $2x+3y+2z=1$.

(10 x 3 = 30)

Part III

(Answer <u>any six</u> questions)

23. Compute **adj A** and A^{-1} for the matrix $A=\begin{bmatrix}3&2&2\\1&3&1\\5&3&4\end{bmatrix}$.

24. Find the condition on a, b and c, so that the following system of equations possess a solution: $x+2y-3z=a$, $3x-y+2z=b$, $x-5y+8z=c$.

25. Find the rank of the matrix $A=\begin{bmatrix}1&2&1\\-1&0&2\\2&1&-3\end{bmatrix}$.

26. Find the radius of curvature at any point (x,y) on the rectangular hyperbola $xy=c^2$.

27. Examine the function x^3+y^3-3axy for maximum value.

28. Evaluate $\int_0^1\int_{y=x}^{\sqrt{x}}(x^2+y^2)\,dx\,dy$.

29. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\, dx\, dy\, dz$.

30. Find $\frac{dz}{dt}$ if $z = xy$ where $x = \cos t$ and $y = \sin t$.

(6 x 5 = 30)

Part IV

(Answer *__any one__* *question)*

31. Diagonalise the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

32. Find the maximum and minimum distance of the point $(3,4,12)$ from the sphere $x^2 + y^2 + z^2 = 1$.

(1 x 10 = 10)