Basc. 1205   Engineering Mathematics II (3+0)

Marks: 50

Time:2 hours

I   **Answer the following**   **(10x1=10)**

1   Discuss the nature of the series $\dfrac{1}{4} + \dfrac{3}{7} + \dfrac{5}{10} + \cdots$

2   State Raabe's test.

3   $\left[y \left(1 + \dfrac{1}{x}\right) + \cos y \right] dx + [x + \log x - x \sin y] dy = 0$ is an example of .... differential equation.

4   Solve $\dfrac{d^3 y}{dx^3} + y = 0$.

5   Find the particular solution for the differential equation $(D^2 + 5D + 6)y = 4^x$.

6   Find A' for the differential equation $y'' + a^2 y = \sec(ax)$, by using the method of variation of parameters.

7   Obtain the complementary function for the differential equation
$x^2 y'' + xy' + 9y = 3x^2 + \sin(3 \log x)$.

8   Form the partial differential equation from the function $z = f(x^2 + y^2)$.

9   Solve $zxp + yzq = xy$.

10   State the formula to solve the equation of the form $f(x, y, z, p, q)$ using Charpit's method.

II   **Write Short notes on any FIVE of the following**   **(5x2=10)**

1   Show that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \sin x$.

2   Solve $[D^4 - 18D^2 + 81] y = 36 \, e^{3x}$.

3   Find the nature of the series $\dfrac{1}{\sqrt{2}} + \dfrac{2}{\sqrt{9}} + \dfrac{3}{\sqrt{28}} + \dfrac{4}{\sqrt{68}} + \cdots$

4   Form a partial differential equation for the function
$z = f(x + ay) + g(x - ay)$, by the method of elimination of arbitrary functions.

5   Solve $\quad x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x + y)z$.

6   Solve $\dfrac{d^2 y}{dx^2} + 3 \dfrac{dy}{dx} + 2y = 4 \cos^2 x$.

7   Solve $\left[2 + 2x^2 \sqrt{y} \right] y \, dx + \left[x^2 \sqrt{y} + 2 \right] x \, dy = 0$.

P.T.O

III   **Answer any FIVE of the following.**   (5x4=20)

1. a. Discuss the convergence of $\dfrac{1^2}{4^2} + \dfrac{1^2.5^2}{4^2.8^2} + \dfrac{1^2.5^2.9^2}{4^2.8^2.12^2} + \cdots$

   b. Using Cauchy's root test, discuss the convergence of the series $\sum_{n=1}^{\infty} \dfrac{(n+1)^n}{n^{n+1}} . x^n$

2. a. Solve $(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$.

   b. Solve $xy(1 + xy^2)\dfrac{dy}{dx} = 1$.

3. Solve the following

   a. $\dfrac{d^2y}{dt^2} - 4\dfrac{dy}{dt} + 13y = e^{3t}\cosh 2t + 2^t$

   b. $[D^3 + 6D^2 + 11D + 6]\, y = 0$

4. Solve for p, if $p^2 + 2py\cot x = y^2$

5. Solve $\dfrac{dy}{dx} + y = z + e^x$.

6. Solve $x^4\dfrac{d^3y}{dx^3} + 2x^3\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} + xy = \sin(\log x)$.

7. a. Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

   b. Solve $p^2 + pqy = z$ by Charpit's method.

IV   **Write an essay on any ONE of the following**   (1x10=10)

1. a. Solve by the method of variation of parameters $(D^2 - 3D + 2)y = \cos(e^{-x})$.

   b. Find the complete integral of $pxy + pq + qy = yz$ by Charpit's method.

   c. Solve $zxp + yzq = xy$.

2. a. Solve $[1 + e^{x/y}]dx + e^{1/y}\left[1 - \dfrac{x}{y}\right]dy = 0$.

   b. Solve $y'' + 4y' - 12y = e^{2x} - 3\sin 2x$.

   c. Discuss the convergence of the series $\dfrac{p}{q} + \dfrac{p(p+a)}{q(q+b)} + \dfrac{p(p+a)(p+2a)}{q(q+b)(q+2b)} + \cdots$

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