KERALA AGRICULTURAL UNIVERSITY

B.Tech (Food. Engg) 2012 Admission

III rd Semester Final Examination- December /January -2013

Cat. No: Basc.2108

Title: Engineering Mathematics III (2+1)

Marks: 80

Time: 3 hours

PART A

(Answer *all* questions, each carries 4 marks)

1. If $\vec{u} = 5t^2\hat{i} + t^3\hat{j} - t\hat{k}$ and $\vec{v} = 2\sin t\hat{i} - \cos t\hat{j} + 5t\hat{k}$ .Find $\frac{d}{dt}(\vec{u}.\vec{v})$

2. If f is a scalar point function then show that $\nabla \times \nabla f = 0$

3. Give the Fourier series for the function $f(x)$ in the interval $\alpha < x < \alpha + 2\pi$, giving the definition of Euler's formulae.

4. Find the image of the circle $|z| = 1$ under the map $w = z + 3 + 2i$

5. State Cauchy's integral theorem

(5 x 4 = 20)

PART B

(Answer any *five* questions, each carries 6 marks)

1. Find the angle between the surfaces $z = x^2 + y^2 - 3$ and $x^2 + y^2 + z^2 = 9$ at the point $(2,-1,2)$

Show that $Div(Grad\ r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$

3. By using the sine series for $f(x) = 1$ in $0 < x < \pi$, show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + ...$

4. Find the Fourier series of $f(x) = x^2 - 2$ in the interval $-2 \leq x \leq 2$

5. Find the bilinear transformation which maps the points $z_1 = -2, z_2 = 0 \& z_3 = 2$ into the point $w_1 = 0, w_2 = i \& w_3 = -i$

6. Find the Laurent's series expansion of $f(z) = \frac{1}{(z-1)(z+3)}$ in the region $1 < |z| < 3$

7. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$; where C is the circle $|z| = 3$

(5 x 6 =30)

PART C

(Answer any three questions, each carries 10 marks)

1. a). State Green's theorem for the plane

   b). Find the area of a circle of radius *'a'* using Green's theorem

2. a).Using Stokes' theorem show that $\oint_C \vec{r}.d\vec{r} = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

   b).Use Gauss Divergence theorem to evaluate $\iint_S \vec{F}.dS$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$

3 .Expand $f(x) = \begin{cases} \frac{1}{4} - x & \text{if} \quad 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{if} \quad \frac{1}{2} < x < 1 \end{cases}$ as the Fourier series of

   a) Sine terms (sine series)

   b) Cosine terms (cosine series)

4. Verify the following functions are harmonic, if so determine the analytic function $f(z) = u + iv$

   a) $u = e^x \cos y$

   b). $u = x^2 - y^2$

5. a).Evaluate $\int_{|z|=3} \frac{e^{2z}}{(z-1)(z-2)} dz$ using Cauchy's integral formula

   b).Using Residue theorem Evaluate $\int_0^\infty \frac{1}{1+x^2} dx$