KERALA AGRICULTURAL UNIVERSITY

B.Tech (Food. Engg) 2011 Admission
III rd Semester Final Examination- January/February -2013

Cat. No: Basc.2108   **Marks: 80**
Title: Engineering Mathematics III (2+1)   **Time: 3 hours**

Part A (answer all questions)

1. Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ where Q is the point $(5,0,4)$.

2. If $\bar{r} = xi + yj + zk$,show that $\text{div } \bar{r} = 3$ and $\text{curl } \bar{r} = 0$.

3. Obtain the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.

4. Explain the transformation $w = z + c$, where c is a complex constant.

5. Calculate the residue of $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ at its simple pole.

(5 x 4=20)

Part B (answer any five)

6. Evaluate $\displaystyle\int_{0}^{1+i} (x^2 - iy)\, dz$ along the path $y = x^2$.

7. Determine the analytic function whose real part is $u = 3x^2 y - y^3$.

8. Expand $\dfrac{1}{(z-1)(z-2)}$ as a series in the region $|z| < 1$.

9. Find the half-range cosine series of $f(x) = (x-1)^2$ in the interval $(0,1)$.

10. Using residue theorem, evaluate $\displaystyle\int_{c} \dfrac{2z-1\ dz}{z(z+1)(z-3)}$ where c is $|z| = 2$.

11. Find the image of the line $y - x + 1 = 0$ under the mapping $w = \dfrac{1}{z}$.

12. Find the Fourier sine transform of $e^{-|x|}$.

(5 x 6=30)

Part C (answer any two)

13. Find the Fourier series representation of $f(x) = \begin{cases} x & \text{in } (0,\pi) \\ 2\pi - x & \text{in } (\pi, 2\pi) \end{cases}$ .Deduce

that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots\ldots = \dfrac{\pi^2}{8}$.

14. a) prove that $\operatorname{div}\left(\phi\bar{f}\right)=\phi\ \operatorname{div}\ \bar{f}+\left(\operatorname{grad}\phi\right)\cdot\bar{f}$,where ϕ is a scalar point function and $\bar{f}$ is a vector point function

b) Prove that $\operatorname{curl}\left(\operatorname{grad}\phi\right)=0$

15. Verify Green's theorem in the plane for $\oint\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy$ where c is the boundary of the region bounded by $x=0, y=0, x+y=1$

(2 x 10=20)

Part D (answer either **a** or **b**)

16. a) Evaluate $\int_c \dfrac{12z-7\ dz}{\left(z-1\right)^2\left(2z+3\right)}$ where c is $\left|z\right|=2$

b) Evaluate $\int_0^{2\pi} \dfrac{d\theta}{2+\cos\theta}$.

(10 x 1=10)