FINITE ELEMENT ANALYSIS – APPLICATIONS IN AGRICULTURAL ENGINEERING

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PROJECT REPORT

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DECLARATION

We hereby declare that this project report entitled **"FINITE ELEMENT ANALYSIS – APPLICATIONS IN AGRICULTURAL ENGINEERING"** is a bonafide record of project work done by us during the course of project and the report has not previously formed the basis for the award to us of any degree, diploma, associate ship, fellowship or other university or society.

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CERTIFICATE

Certified that this project work entitled **"FINITE ELEMENT ANALYSIS – APPLICATIONS IN AGRICULTURAL ENGINEERING"** is a record of project work done jointly by **Greeshma, C. P., Muhammad, A., and Shincy Lukose** under my guidance and supervision and it has not previously formed the basis for the award of any degree, diploma, fellowship or associate ship to them.

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SYMBOLS AND ABBREVIATIONS

et al.	and others
KAU	Kerala Agricultural University
KCAET	Kelappaji College of Agricultural Engineering and Technology
k	coefficient of permeability
FEM	Finite Element Method
F	recharge
m	Metre
s	seconds
u	piezometric head
w	weight function
∇	Gradient operator
2-D	Two-dimensional
3-D	Three-dimensional
∂	Partial derivative
Ω	Region of the problem
Γ	Boundary of theproblem
n_x	component of unit normal vector along X-direction
n_y	component of unit normal vector along Y-direction

q_n	secondary variable of formulation
K_{ij}	influence coefficient

CHAPTER I

INTRODUCTION

The finite element method (FEM) is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. Although originally developed to study stresses in complex airframe structures, it has since been extended and applied to the broad field of continuum mechanics. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering schools and in industry.

Agricultural engineering is the combination of almost all basic engineering disciplines with its orientation in technology implementation in agriculture. Being a comparatively new branch of technology, agricultural engineering requires the application of modern science and technology for the growth and development of agriculture. The scope of agricultural engineering is both wide and varied, covering such diversified field as farm machinery and power, agricultural processing, soil and water conservation along with farm irrigation, ground water, farm buildings and structures, and environmental engineering. The application of finite element analysis is comparatively new in the area of agricultural engineering, so the utilisation of Finite Element is very much demanding in the current scenario for solving the complex problems related to Agricultural engineering.

FEM is a well-established numerical technique, which has been used to obtain the numerical solution of various real life problems in the areas of engineering and science. The method has been developed over about thirty years, from the traditional matrix methods for structural analysis for frames and trusses. In many cases, finite-element models show a more realistic spatial discretization than finite-difference models.

FEM software provides a wide range of simulation options for controlling the complexity of both modelling and analysis of a system. This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications. The introduction of FEM has substantially decreased the time taken by products from concept to the production line. Initial prototype designs using FEM has accelerated and improved testing and development. FEM improves accuracy, enhance the design and give better insight into the critical design parameters, leading to a virtual prototyping and fewer hardware prototypes. The result is a faster and less expensive design cycle, increased productivity and increased revenue.

Movement of groundwater in the subsurface is responsible for a variety of environmental and geological processes including heat transfer and solute transport. The groundwater is extracted from aquifers through pumping wells and supplied to domestic use, industry and agriculture. With increased withdrawal of ground water, the quality of ground water has been continuously deteriorating. Also water can be injected into aquifers for storage and/or quality control purposes. Increased demand for water has stimulated development of techniques for investigating the occurrence and movement of groundwater. So there is a need to evaluate them and mathematical modelling provides an essential quantitative tool. One of the important developments in groundwater hydrology in recent years has been the introduction of numerical groundwater models. These have made an improved understanding of complex groundwater systems possible. In recent years, the demand for using computer simulations increased to make predictions of flow and transport in the subsurface, thus familiarity with the fundamental principles behind modelling is critical.

It is evident from a study of the journals, proceedings and research reports that the use of groundwater modelling has increased over the past 15 years. Numerical modelling is becoming an increasingly important tool for analysing complex problems involving water flow and contaminant transport in the unsaturated zone.

An understanding of ground water flow modelling helps to

- Determine the total volume that can be withdrawn annually from the aquifer.
- Make Decisions related to groundwater quality.
- Study the effect of Agricultural activities such as the use of fertilizers and pesticides.
- Identify the best potential locations for future well installations and show how the pumping of one or more wells will affect the other wells in the aquifer and surface water bodies.

. Heat transfer analysis is a problem of major significance in a vast range of industrial applications. These extend over the fields of mechanical engineering, aeronautical engineering, agricultural engineering, chemical engineering and numerous applications in civil and electrical engineering. Thermal simulation play an important role in the design of many engineering applications including thermal combustion engines turbines, heat exchangers, piping systems, electronic components etc. Advanced topics which include features such as phase change, coupled heat and mass transfer, and thermal stress analysis provides the engineer with the capability to address a further series of key engineering problems. The complexity of practical problems is such that closed form solutions are not generally possible. The use of numerical techniques to solve such problems is therefore considered essential. The capabilities of FEM can be easily expanded to several applications within the food industry. The application of heat and mass transfer to drying problems and the calculation of both thermal and shrinkage stresses can be modeled using finite element method. Finite element models are often used to study the heat transfer characteristics of a device, to understand where and how heat is rejected as well as the transient and steady-state temperature distributions.

The FEM can be used in food industry to:

- Study cold and hot spots within various foods of irregular geometries
- Predict the microbial destruction and nutrient degradation
- Optimize the process
- Develop information and nomograms to educate the consumer on heating techniques for food safety and quality

FEA is a widely accepted computer simulation methodology for modelling, evaluating, and optimizing farm machinery equipment /tool’s mechanical and structural design. The technology has a long history of effective use in the energy industry as well as in the agricultural engineering and automotive sectors. Engineers generally start with a CAD model and then use FEA software to transform that model into a 3D mesh of geometric units – the ‘elements’ in finite element analysis. FEA is capable of reducing design time as well as the expense of extensive physical prototyping. The technology has been accelerated in recent years with the addition to the engineer’s toolbox of process integration and design optimization software, as well as multi-core, high-performance computing. Extensive FEA materials libraries help the design engineer to model and predict the response of different materials under such huge temperature and/or pressure differentials. Subsoilers work in the very arduous conditions, so they bear heavy dynamic loads.

Design optimization software, used in combination with FEA, enables engineers to automate the exploration of multiple design alternatives and arrive at design performance answers faster and with higher degree of confidence than is possible with manual analysis techniques alone. This leads to improve the strength of our design. ANSYS is a general purpose software package based on the finite element analysis. This allows full three-dimensional simulation without compromising the geometrical details.

OBJECTIVE OF PRESENT STUDY

- Study of finite element method
- Mathematical formulation of two-dimensional problems governed by Poisson's equation
- Numerical implementation of two-dimensional problems related to agricultural engineering
- Analysis of some agricultural engineering related problems using the finite element software package ANSYS

CHAPTER II

REVIEW OF LITERATURE

The finite-element method originated from the need for solving complex elasticity and structural analysis problems in civil and aeronautical engineering. Its development can be traced back to the works by Alexander Hrennikoff (1941) and Richard Courant (1942). The approaches used by them were different, but share one essential characteristic, discretization of a continuous domain into a set of discrete sub-domains, usually called elements. Hrennikoff's work discretizes the domain by using a lattice analogy while Courant's approach divides the domain into finite triangular sub-regions for solution of second order elliptic partial differential equations that arise from the problem of torsion of a cylinder.

Development of the finite element method began in the middle of 1950s for airframe and structural analysis and progressed through the work of John Argyris and Ray W. Clough in the 1960s for use in civil engineering. By late 1950s, the key concepts of stiffness matrix and element assembly existed essentially in the form used today and NASA issued a request for proposals to develop the finite element software NASTRAN in 1965. The method was provided with a rigorous mathematical foundation in 1973 with the publication of Strang and Fix's An Analysis of The Finite Element Method, and has since been generalized into a branch of applied mathematics for numerical modelling of physical systems in a wide variety of engineering disciplines. Regarding the application of FEM in hydrology, though the formulation of the problem can be found as early as 1966 by Zienkiewicz, the method has a slow progress for ground water flow problems.

GROUND WATER

Mosé *et al.*, (1994) studied the groundwater flow problem with a two-way comparison between the mixed hybrid finite element method and the standard finite element method (also called the conforming finite element method). The simulations were presented for two-dimensional case with a triangular space discretization because of its practical interest for hydro-geologists. The results of the simulations were presented in the form of streamlines. The comparison studies showed that the mixed hybrid finite element is superior to the conforming method in terms of accuracy and computational effort. The potential fields obtained by the mixed hybrid and the conforming finite element methods were the same.

Rabbani (1994) reported that real-world groundwater modelling dealt with heterogeneous and anisotropic aquifer systems. He developed a new approach to overcome the limitations of the conventional finite element method. In that approach, a two-dimensional model was discretized into triangular elements. Element matrices were formed directly based on flow principles and material properties of elements, without using shape functions and integration process.

Yu *et al.,* (1994) modeled three dimensional groundwater flow by Modified Finite-Element Method. In this, a number of theoretical improvements were made to the finite-element formulation for modelling three-dimensional steady and unsteady ground-water flow. The Galerkin method was combined with the collocation method to handle the time-derivative term of the governing equation and the resulting system of ordinary differential equations was solved by using finite integration.

Larabi *et al.,* (1997) developed a numerical procedure for solving 3-D phreatic groundwater flow with saltwater intrusion. The model used a fixed finite-element (FE) mesh, and iteratively adjusted the water table and the saltwater-interface positions by excluding flow in the unsaturated zone and the saltwater zone. Validation of the numerical model was further made with respect to observations from a 3-D laboratory sand box involving phreatic flow coupled with a saltwater interface. They obtained model simulations and experimental results in good agreement and claims that this model can also be used for groundwater management.

Janssen (2004) presented a study of analytical and numerical models for groundwater flow calculations. The numerical models were found to be more effective for incorporating various subsoil and boundary conditions in detail.

Dawson *et al.,* (2006) studied locally conservative, stabilized finite element methods for variably saturated flow. Standard Galerkin finite element methods for variably saturated groundwater flow were found to have several deficiencies. They considered conforming finite element discretizations based on a multiscale formulation along with recently developed, local post processing schemes. Accuracy and efficiency of the proposed schemes were evaluated through a series of steady-state and transient variably saturated groundwater flow problems in homogeneous as well as heterogeneous domains.

Bibin Sunny *et al.,* (2009) studied the finite element method as a part of their B.Tech degree program. They developed a program in visual C++ to model the groundwater flow. And also they analyzed different 1-D and 2-D numerical groundwater flow problems.

HEAT TRANSFER

Jiri Kratochvil *et al.,* (1989) studied the embankment failure due to the destabilizing effect of seepage forces of the infiltrating water during floods. The analogy between seepage and heat diffusion is used to analyze the hydraulic problem with ANSYS/THERMAL. The results of the numerical solution using ANSYS/THERMAL was compared with experiments carried out in the hydraulic flume. The numerical results obtained proved that the ANSYS/THERMAL is a powerful instrument for the detailed analysis of the transient hydrodynamic fields in embankment dams during flood periods.

Lin *et al.,* (1995) used the finite element method to predict temperature distribution in agar gels by using the commercial software TWODEPEP. It has been used to model microwave processing of foods.

Ozkan Sarikaya *et al.,* (2005) investigated the heat transfer characteristics for different ceramic coatings in thermal barrier applications using finite element method (FEM). The effect of the different types of coatings on thermal insulation properties and residual stresses was discussed based on the results by FEM. It was evaluated that the best thermal barrier coating systems, which have the lowest residual stresses and high temperature difference, were determined on the interlayer and bond coat of the MgO–ZrO2 coating system with five layers, and also finite element technique used to optimize the heat transfer characteristics of the thermal barrier ceramic coatings

Vittorio Raffaele *et al.,* (2005) analysed microwave heating of foodstuff, characterised by cylindrical geometry, using a finite element model. The model has been set and solved by a commercial package, FEMLAB.

Stefano *et al.,* (2008) studied the behaviour of a cylindrical-shaped vegetable sample in a drier, by using finite elements method. An experimental study was undertaken which showed very good agreement between model predictions and experimental results. The proposed model was found useful for the situations in which, either semi-empirical correlation were not currently available or operating conditions were changed during drying process. The model could determine which set of operating conditions would enhance the quality and the safety of the final product.

Hikmet Esen et al., (2009) studied temperature distributions in boreholes of a vertical ground-coupled heat pump system (GCHP). A two-dimensional finite element model (FEM) was developed to simulate temperature distribution development in the soil surrounding the Ground Heat Exchanger (GHE) of GCHP operating in the cooling and the heating modes. The model was shown to be very compatible for showing the temperature distribution development in the boreholes with time. Presently, FEM appears to be most promising for predicting the response of GHEs to thermal loading.

FARM MACHINERY

Kushwaha *et al.,* (1990) applied finite element analysis to study the effect of friction coefficient on the tool draft. Laboratory tests were conducted to investigate the soil forces on a metallic-glass-coated cultivator sweep, and these forces were compared with the forces on a regular sweep. The results from the theoretical analysis and the experimental tests showed a significant decrease in draft with the reduction in friction coefficient between the soil and the tool surface.

Gupta and Maheshwari (1992) analyzed the stress over a cultivator shovel moving at a certain depth. The assumptions made it possible to apply the theory of bending of laterally loaded plates for the computation of stresses set up in the tillage tool. This theory involves fundamental equations of elasticity, namely the equations of equilibrium, equations of continuity and surface conditions. The theoretically calculated values of stresses were fairly close to experimentally observed values.

Brown *et al.,* (1998) outlined a stress analysis performed on a Chisel Plow using finite element analysis. A proposed redesign of the hitch permitted full 360-deg front wheel caster and resulted in lower stress levels within the members of the hitch.

Nidal and Randall (2003) done Nonlinear 3D Finite Element Analysis of the Soil Forces Acting on a Disk Plow. The study aimed to compare predicted soil forces on a disk plow with measured forces within the tillage depth of clay and sandy loam soils. The model assumed the effects of both tilt angle and plowing speed. A 3D nonlinear finite element model was used to predict the soil forces while a dynamometer was used to measure them on a disk plow in the field.

Gebregziabher *et al.,* (2007) validated design of the Ethiopian plough using structural analysis with finite element analysis. The force analysis was validated by means of the finite element (FE) analysis using the ABAQUS package. They confirmed that draught force on the ploughshare increased with pulling angle. The output of the FEM and traditional calculation resulted in small errors of less than 3% for draught and 5% for vertical forces.

Kaveh Mollazade *et al.,* (2010) modelled subsoilers with various shapes like C shape, sloping shape, and L shape in order to choose best one of them with maximum working life. Clay loam soil condition was used as a tool to find the value of soil resistance forces and models were analyzed with ANSYS software. Results showed that shape of subsoiler has no significant roll in the maximum number of allowable force exertion cycles which caused to fracture of subsoiler's shank. It also showed that C shape has better design than the others and this makes the higher factor of safety for C shape subsoiler and consequently have more working life.

CHAPTER III

MATERIALS AND METHODS

The governing equation for two-dimensional groundwater flow in homogenous soil and heat transfer in an isotropic region is in the form of Poisson's equation. In finite element method, the given domain is divided into sub-domains, called finite elements, and an approximate solution to the problem is developed over the domain. The methods used and methodology adopted for Finite Element Analysis of two-dimensional problems is described in this chapter.

3.1 MATHEMATICAL FORMULATION

3.1.1 Finite Element Method

The finite element method is a numerical technique that employs the philosophy of constructing piecewise approximations of solutions to problems described by differential equations. The finite element method is a powerful technique devised to numerically evaluate the complex field problems. The most distinctive feature of the finite element method, that separates it from other methods, is the division of a given domain into a set of simple sub domains, called finite elements. The finite element method overcomes the disadvantages of the traditional variational methods by providing a systematic procedure for the derivation of the approximation functions over sub regions of the domain.

Thus, the finite element method can be viewed, in particular, as an element wise application of the weighted residual method. In it, the approximation functions are often taken to be algebraic polynomial, and undetermined parameters represent the values of the solution at a finite number of pre-selected points, called nodes, on the boundary and in the interior of the element.

Since the finite element method is a technique for constructing approximation functions required in an element based application of any variational method, it is necessary to study the weighted-integral formulation and to arrive at the weak form of differential equations. The primary objectives will be to construct the weak form of a given differential equation and to classify the boundary conditions associated with the equation. A weak form is weighted –integral statement of a differential equation in which the differentiation is distributed among the dependent variable and the weight function and includes the natural boundary conditions of the problem.

There are three steps in development of the weak form of any differential equations. In the first, we put all expressions of the differential equation on one side (so that the other side is equal to zero), then multiply the entire equation by a weight function and integrate over the domain of the problem. The resulting expression is called the weighted-integral form of the equation. In the second step, we use integration by parts to distribute differentiation evenly among the dependent variable and the weight function, and use the boundary terms to identify the form of primary and secondary variables. This is the weak form or variational form of the governing equation from which we form the finite element equations. In the third step, we modify the boundary terms by restricting the weight function to satisfy the homogeneous form of the specified essential boundary conditions and replacing the secondary variables by their specified values.

3.1.2 Steps involved in the finite element method

i. Discretization of the given domain in to a collection of pre-selected finite elements. The size of the elements can be uniform or non-uniform. Each element is assumed to be connected to the neighbouring elements at nodes. The collection of elements is called finite element mesh. Generate the geometric properties (coordinates, cross-sectional area etc.) of the element needed for the problem.

ii. Derivation of elements equation for typical elements in the mesh.

Construct a variational form of the differential equations over the domain of the problem. Assume that the variable of the problem is of the form $u = u_1N_1 + u_2N_2$ where u_1 and u_2 are the nodal values of the problem and N_1 and N_2 are the assumed shape functions or interpolation functions. Select element interpolation functions or shape function and formulate the element equation of the problem in the form $[K^e]\{u^e\} = \{F^e\}$ where K^e is the matrix containing the influence coefficients, u^e the nodal displacements and F^e the nodal force of the element.

iii. Assembly of element equations to obtain the equation of the whole problem.

iv. The element equations are assembled after identifying the inter-element continuity conditions between the local and global degrees of freedom. The boundary conditions are then imposed.

v. Solution of the assembled equations.

The assembled equations are solved to get the nodal unknowns of the problem.

vi. Post processing of the result.

Compute the variables of the problem at desired points other than nodes from the nodal values. Represent the results in a tabular or graphical form.

3.1.3 Formulation of two-dimensional ground water flow and heat transfer under steady state condition

The governing equation for two-dimensional ground water flow in homogeneous soil and heat transfer in an isotropic rectangular region is in the form of the Poisson equation given by

$$k\nabla^2\phi + f = 0 \qquad (1)$$

where

k = coefficient of permeability / thermal conductivity

f = recharge / internal heat generation

u = piezometric head / temperature and

∇ = gradient operator

The gradient operator can be expressed as

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y}$$

Where *i* and *j* denotes the unit vectors directed along the *x* and *y* axes respectively. Then, the Eq. (1) can be written as

$$k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f = 0 \qquad (2)$$

For the development of weak form, consider an arbitary element Ω^e, whether triangular or quadrilateral, of the finite element mesh. Development of weak form over Ω^e include three steps. The first step is to multiply Eq. (2) with a weight function 'ω', which is assumed to be differentiable once with respect to *x* and *y*, and then integrate the resulting equation over the element domain Ω^e.

$$k\int_{\Omega^e}\omega\left[\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + f\right]dxdy = 0 \qquad (3)$$

In the second step, the differentiation is equally distributed between *u* and *w*. To achieve this integrate the first two terms in Eq. (3) by parts

$$\frac{\partial}{\partial x}\left(w\frac{\partial u}{\partial x}\right) = \frac{\partial w}{\partial x}\frac{\partial u}{\partial x} + w\frac{\partial^2 u}{\partial x^2} \tag{4}$$

$$w\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(w\frac{\partial u}{\partial x}\right) - \frac{\partial w}{\partial x}\frac{\partial u}{\partial x} \tag{5}$$

$$\frac{\partial}{\partial y}\left(w\frac{\partial u}{\partial y}\right) = \frac{\partial w}{\partial y}\frac{\partial u}{\partial y} + w\frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$w\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(w\frac{\partial u}{\partial y}\right) - \frac{\partial w}{\partial y}\frac{\partial u}{\partial y} \tag{7}$$

Using divergence theorem

$$\int_{\Omega^e} \frac{\partial}{\partial x}\left(w\frac{\partial u}{\partial x}\right) dxdy = \oint_{\Gamma^e} \frac{\partial u}{\partial x} n_x ds \tag{8}$$

$$\int_{\Omega^e} \frac{\partial}{\partial y}\left(w\frac{\partial u}{\partial y}\right) dxdy = \oint_{\Gamma^e} \frac{\partial u}{\partial y} n_y ds \tag{9}$$

where n_x and are component of unit normal vector along *x* and *y* coordinates.

Applying equations (5), (7), (8) and (9) in equation (3) we get the weak form as

$$\int_{\Omega^e} \left(-\frac{\partial w}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial y} + wf \right) dxdy + \oint_{\Gamma^e} \left(\frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y \right) ds = 0 \tag{10}$$

Specifying the coefficient of the weight function in the boundary expression

$$q_n = \frac{\partial u}{\partial x} n_x + \frac{\partial u}{\partial y} n_y \tag{11}$$

constitutes the natural boundary condition and q_n is the secondary variable of the formulation.

The third and last step of the formulation is to use Eq. (11) in Eq. (10) to get the weak form of Eq. (3) as

$$\int_{\Omega^e} \left(-\frac{\partial w}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial y} + wf \right) dxdy + \oint_{\Gamma^e} q_n w ds = 0 \tag{12}$$

Suppose that *u* is approximated over the finite element by the expression

$$u(x, y) = \sum_{j=1}^{n} u_j^e N_j^e (x, y) \tag{13}$$

Where u_j^e is the value of u at j^{th} node. Substituting equation (13) into (12)

$$k\int_\Omega \left[\frac{\partial w}{\partial x}\sum_{j=1}^{n}\frac{\partial N_j}{\partial x} - \frac{\partial w}{\partial y}\sum_{j=1}^{n}\frac{\partial N_j}{\partial y} + wf \right] dxdy + \oint_\Gamma q_n w ds = 0 \qquad (14)$$

Since we need *n* independent algebraic equations to solve for the n unknowns $u_1,, u_3 \;...u_n$, we choose *n* independent functions for *w*: $w = N_1, N_2, \ldots\ldots N_n$

The i[th] algebraic equation is obtained by substituting $w = N_i$ into equation (12)

$$\sum_{j=1}^{n}\left\{\int_\Omega k\left(\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}\right) dxdy\right\} = \int_\Omega f N_i dxdy + \oint_\Gamma q_n N_i ds \qquad (15)$$

or $$\sum_{j=1}^{n} K_{ij}^e u_j^e = f_i^e + Q_i^e \qquad (16)$$

In matrix notation this takes the form

$$\left[K^e\right]\left\{u^e\right\} = \left\{f^e\right\} + \left\{Q^e\right\} \qquad (17)$$

If we know the head at each node $u_1,, u_3 \;...\; u_n$, we can determine the head or temperature within the element.

The element displacement vector $\left\{u^e\right\}$ for the triangular element is given

$$\left\{u^e\right\} = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \qquad (18)$$

The element nodal displacement u(x) at any interior point of the element by the following linear polynomial in x

u(x)=a + bx + cy (19)

In matrix form, the conditions are written as

$$u(x) = \begin{bmatrix} 1 & x & y \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \qquad (20)$$

Or $\{u\} = [x]\{a\}$ (21)

The unknown coefficients a, b, c are determined based on the following conditions at the nodes.

At node 1, $x = x_{1,}\; y = y_1 \; and \; u = u_1$

At node 2, $x = x_2, y = y_2$ and $u = u_2$

At node 3, $x = , y = y_3$ and $u = u_3$

$$u_1 = u(x_1, y_1) = a + bx_1 + cy_1 \tag{22}$$

$$u_2 = u(x_2, y_2) = a + bx_2 + cy_2 \tag{23}$$

$$u_3 = u(x_3, y_3) = a + bx_3 + cy_3 \tag{24}$$

$$\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{bmatrix} \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} \tag{25}$$

Or $\{u^e\} = [A]\{a\}$ (26)

Hence we get the coefficient vector $\{a\}$

$$\{a\} = [A]^{-1}\{u^e\} \tag{27}$$

On the other hand the interpolation process can be represented by

$$\{u\} = [N]\{u^e\} \tag{28}$$

From equations (23) and (25) we have

$$\{u\} = [x]\{a\} = [x][A]^{-1}\{u^e\} \tag{29}$$

Comparing equation (27) with equation (26), we see that

$$[N] = [x][A]^{-1} \tag{30}$$

Where the elements of the matrix $[N]$ are denoted by N_1, N_2, N_3 These are linear interpolation polynomials in x and y, and are obtained as

$$N_i = \frac{1}{\Delta(a_i + b_i x + c_i y)}, \; i = 1,\; 2,\; 3. \tag{31}$$

Where in which *i* , *j* and *k* are to be taken in cyclic order. Δ is given by

$$\Delta = \begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \text{ 2x area of triangle}$$

$$a_1 = x_2 y_3 - x_3 y_2 \quad a_2 = x_3 y_1 - x_1 y_3 \quad a_3 = x_1 y_2 - x_2 y_1$$

$$b_1 = y_2 - y_3 \qquad b_2 = y_3 - y_1 \qquad b_3 = y_1 - y_2$$

$$c_1 = x_3 - x_2 \qquad c_2 = x_1 - x_3 \qquad c_3 = x_2 - x_1$$

$$N_1 = \frac{1}{\Delta}\left(\left(x_2 y_3 - x_3 y_2 \right) + \left(y_2 - y_3 \right) x + \left(x_3 - x_2 \right) y \right)$$

$$N_2 = \frac{1}{\Delta}\left(\left(x_3 y_1 - x_1 y_3 \right) + \left(y_3 - y_1 \right) x + \left(x_1 - x_3 \right) y \right)$$

$$N_3 = \frac{1}{\Delta}\left(\left(x_1 y_2 - x_2 y_1 \right) + \left(y_1 - y_2 \right) x + \left(x_2 - x_1 \right) y \right)$$

$$\frac{\partial N_1}{\partial x} = \frac{1}{\Delta}\left(y_2 - y_3 \right)$$

$$\frac{\partial N_2}{\partial x} = \frac{1}{\Delta}\left(y_3 - y_1 \right)$$

$$\frac{\partial N_3}{\partial x} = \frac{1}{\Delta}\left(y_1 - y_2 \right)$$

$$\frac{\partial N_1}{\partial y} = \frac{1}{\Delta}\left(x_3 - x_2 \right)$$

$$\frac{\partial N_2}{\partial y} = \frac{1}{\Delta}\left(x_1 - x_3 \right)$$

$$\frac{\partial N_3}{\partial y} = \frac{1}{\Delta}\left(x_2 - x_1 \right)$$

The interpolation matrix [N] is given by

$$\left[N_1 \right] = \left[N_1 \quad N_2 \quad N_3 \right]$$

$$k_{11} = \iint \left(\frac{\partial N_1}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_1}{\partial y} \right) dA$$

$$k_{12} = \iint \left(\frac{\partial N_1}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_2}{\partial y} \right) dA$$

$$k_{13} = \iint \left(\frac{\partial N_1}{\partial x}\frac{\partial N_3}{\partial x} + \frac{\partial N_1}{\partial y}\frac{\partial N_3}{\partial y} \right) dA$$

$$k_{21} = \iint \left(\frac{\partial N_2}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial y}\frac{\partial N_1}{\partial y} \right) dA$$

$$k_{22} = \left[\frac{\partial N_2}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_2}{\partial y}\frac{\partial N_2}{\partial y} \right] dA$$

$$k_{23} = \left[\frac{\partial N_2}{\partial x}\frac{\partial N_3}{\partial x} + \frac{\partial N_2}{\partial y}\frac{\partial N_3}{\partial y} \right] dA$$

$$k_{31} = \left[\frac{\partial N_3}{\partial x}\frac{\partial N_1}{\partial x} + \frac{\partial N_3}{\partial y}\frac{\partial N_1}{\partial y} \right] dA$$

$$k_{32} = \left[\frac{\partial N_3}{\partial x}\frac{\partial N_2}{\partial x} + \frac{\partial N_3}{\partial y}\frac{\partial N_2}{\partial y} \right] dA$$

$$k_{33} = \left[\frac{\partial N_3}{\partial x}\frac{\partial N_3}{\partial x} + \frac{\partial N_3}{\partial y}\frac{\partial N_3}{\partial y} \right] dA$$

$$k_e = \begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix}$$

$$k_e U = Q$$

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} \\ K_{21} & K_{22} & K_{23} \\ K_{31} & K_{32} & K_{33} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \end{Bmatrix} = Q$$

The element coefficient matrix and element load vectors for various elements are added together considering the local and global degrees of freedom to arrive at a system of linear algebraic equations as given by $[K]\{U\} = \{R\}$

The boundary conditions are imposed and the algebraic equations are solved for the unknown

3.1.4 NUMERICAL IMPLEMENTATION

To investigate the effectiveness of the present formulation we have studied several two-dimensional problems related to agricultural engineering in the fields of groundwater flow and heat transfer. An object oriented computer program in visual C^{++} was developed for the analysis of problems governed by Poisson's equation by using the finite element modelling. A system of linear algebraic equations was obtained for the primary unknowns, temperature in heat transfer problem and piezometric head in groundwater problems, by applying the known boundary conditions. Two numerical examples were solved using the program developed.

3.1.5 Numerical problems:

a) Two dimensional heat transfer.

A steady-state heat conduction in an isotropic rectangular region is considered as the first problem and is of dimensions 15×10 (see Fig.). The origin of the x and y coordinates is taken at the lower left corner such that x is parallel to the side 15 and y is parallel to the side 10. The boundaries x=0 and y=0 are insulated, the boundary x=15 is maintained at zero temperature, and the boundary y=10 is maintained at a temperature $T = T_0 \cos\left(\frac{\pi x}{30}\right)$. The region is discretised using triangular elements of different sizes. The nodal temperatures obtained using various finite element meshes (3×2), (6×4) and (12×8) were compared with analytical solution. The exact solution for the boundary condition (Reddy, 1993) is

$$T(x,y) = T_0 \frac{\cosh\left(\frac{\pi y}{30}\right)}{\cosh \frac{1}{3}\pi} \cos\left(\frac{\pi x}{30}\right)$$

Triangular element mesh (3×2) .

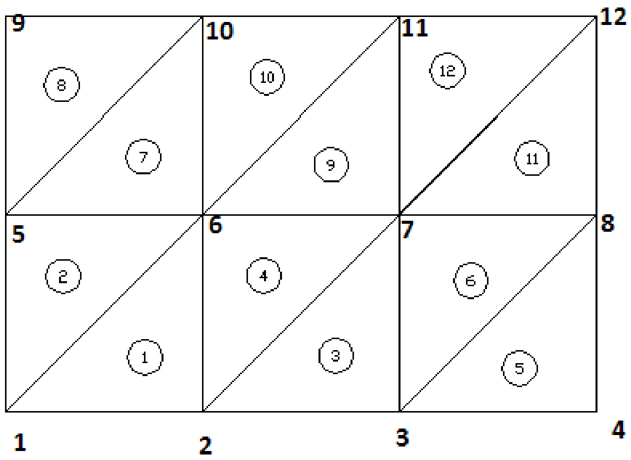

Fig. 1 Geometry with triangular element mesh (3x2)

The given area was divided into 12 nodes with 12 triangular elements of equal edge length.

Program input:

Number of nodes = 12

Number of elements =12

Node	X Co-ordinate	Y Co-ordinate	Node	X Co-ordinate	Y Co-ordinate
1	0	0	7	10	5
2	5	0	8	15	5
3	10	0	9	0	10
4	15	0	10	5	10
5	0	5	11	10	10
6	5	5	12	15	10

1

6

4 1 0

8 1 0

12 1 0

10 1 0.867

9 1 1

11 1 0.5

0

Triangular element mesh (6�4) .

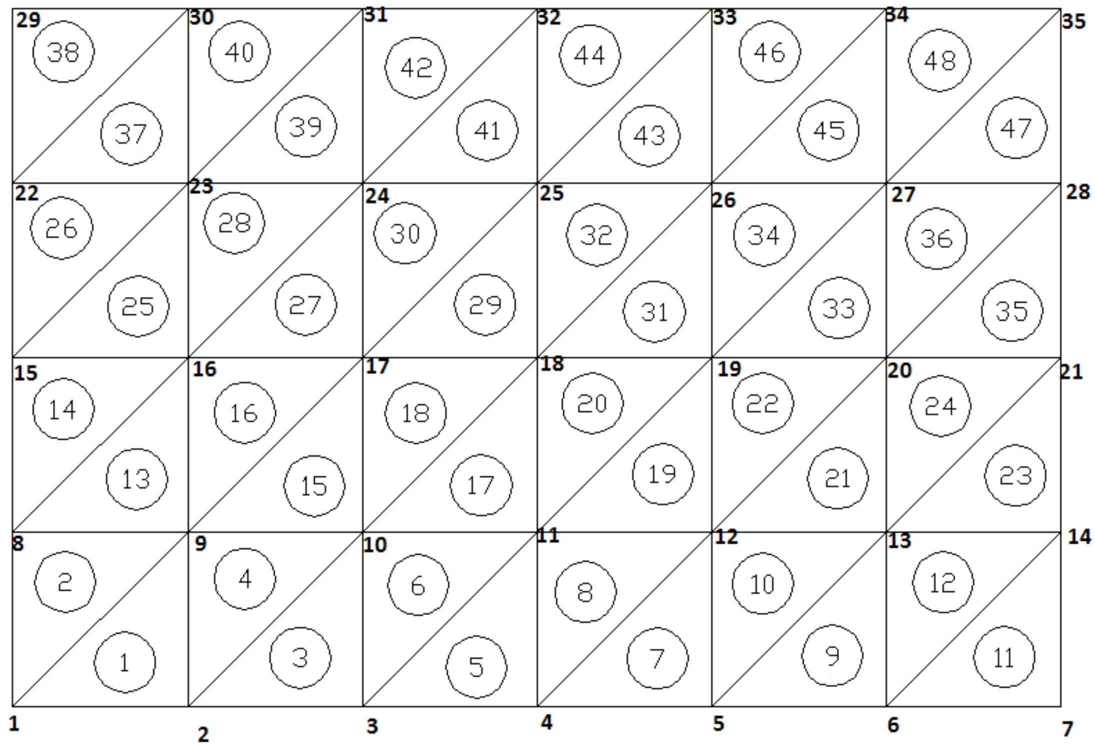

Fig. 2 Geometry with triangular element mesh (6x4)

The given area was divided into 35 nodes with 48 triangular elements of equal edge length.

Program input

Number of nodes = 35

Number of elements =48

Node	X-co ordinate	Y-co ordinate	Node	X-co ordinate	Y-co ordinate
1	0	0	19	10	5
2	2.5	0	20	12.5	5
3	5	0	21	15	5
4	7.5	0	22	0	7.5
5	10	0	23	2.5	7.5
6	12.5	0	24	5	7.5
7	15	0	25	7.5	7.5
8	0	2.5	26	10	7.5
9	2.5	2.5	27	12.5	7.5
10	5	2.5	28	15	7.5

11	7.5	2.5	29	0	10
12	10	2.5	30	2.5	10
13	12.5	2.5	31	5	10
14	15	2.5	32	7.5	10
15	0	5	33	10	10
16	2.5	5	34	12.5	10
17	5	5	35	15	10
18	7.5	5			

1

Element	Node-1	Node-2	Node-3	Element	Node-1	Node-2	Node-3
1	1	2	9	25	15	16	23
2	1	9	8	26	15	23	22
3	2	3	10	27	16	17	24
4	2	10	9	28	16	24	23
5	3	4	11	29	17	18	25
6	3	11	10	30	17	25	24
7	4	5	12	31	18	19	26
8	4	12	11	32	18	26	25
9	5	6	13	33	19	20	27
10	5	13	12	34	19	27	26
11	6	7	14	35	20	21	28
12	6	14	13	36	20	28	27
13	8	9	16	37	22	23	30
14	8	16	15	38	22	30	29
15	9	10	17	39	23	24	31
16	9	17	16	40	23	31	30
17	10	11	18	41	24	25	32
18	10	18	17	42	24	32	31
19	11	12	19	43	25	26	33
20	11	19	18	44	25	33	32
21	12	13	20	45	26	27	34
22	12	20	19	46	26	34	33
23	13	14	21	47	27	28	35

24	13	21	20	48	27	35	34

11

7 1 0

14 1 0

21 1 0

28 1 0

35 1 0

29 1 1

30 1 0.965

31 1 0.866

32 1 0.707

33 1 0.5

34 1 0.258

0

Triangular element mesh (12×8)

The given area was divided into 117 nodes with 192 triangular elements of equal edge length.

Number of nodes =117

Number of elements =192

Node	X co-ordinate	Y co-ordinate	Node	X co-ordinate	Y co-ordinate
1	0	0	60	8.75	5
2	1.25	0	61	10	5
3	2.5	0	62	11.25	5
4	3.75	0	63	12.5	5
5	5	0	64	13.75	5
6	6.25	0	65	15	5
7	7.5	0	66	0	6.25
8	8.75	0	67	1.25	6.25
9	10	0	68	2.5	6.25

10	11.25	0	69	3.75	6.25
11	12.5	0	70	5	6.25
12	13.75	0	71	6.25	6.25
13	15	0	72	7.5	6.25
14	0	1.25	73	8.75	6.25
15	1.25	1.25	74	10	6.25
16	2.5	1.25	75	11.25	6.25
17	3.75	1.25	76	12.5	6.25
18	5	1.25	77	13.75	6.25
19	6.25	1.25	78	15	6.25
20	7.5	1.25	79	0	7.5
21	8.75	1.25	80	1.25	7.5
22	10	1.25	81	2.5	7.5
23	11.25	1.25	82	3.75	7.5
24	12.5	1.25	83	5	7.5
25	13.75	1.25	84	6.25	7.5
26	15	1.25	85	7.5	7.5
27	0	2.5	86	8.75	7.5
28	1.25	2.5	87	10	7.5
29	2.5	2.5	88	11.25	7.5
30	3.75	2.5	89	12.5	7.5
31	5	2.5	90	13.75	7.5
32	6.25	2.5	91	15	7.5
33	7.5	2.5	92	0	8.75
34	8.75	2.5	93	1.25	8.75
35	10	2.5	94	2.5	8.75
36	11.25	2.5	95	3.75	8.75
37	12.5	2.5	96	5	8.75
38	13.75	2.5	97	6.25	8.75
39	15	2.5	98	7.5	8.75
40	0	3.75	99	8.75	8.75
41	1.25	3.75	100	10	8.75
42	2.5	3.75	101	11.25	8.75
43	3.75	3.75	102	12.5	8.75

44	5	3.75	103	13.75	8.75
45	6.25	3.75	104	15	10
46	7.5	3.75	105	0	10
47	8.75	3.75	106	1.25	10
48	10	3.75	107	2.5	10
49	11.25	3.75	108	3.75	10
50	12.5	3.75	109	5	10
51	13.75	3.75	110	6.25	10
52	15	3.75	111	7.5	10
53	0	5	112	8.75	10
54	1.25	5	113	10	10
55	2.5	5	114	11.25	10
56	3.75	5	115	12.5	10
57	5	5	116	13.75	10
58	6.25	5	117	15	10
59	7.5	5			

1

Element	Node-1	Node-2	Node-3	Element	Node-1	Node-2	Node-3
1	1	2	15	97	53	54	67
2	1	15	14	98	53	67	66
3	2	3	16	99	54	55	68
4	2	16	15	100	54	68	67
5	3	4	17	101	55	56	69
6	3	17	16	102	55	69	68
7	4	5	18	103	56	57	70
8	4	18	17	104	56	70	69
9	5	6	19	105	57	58	71
10	5	19	18	106	57	71	70
11	6	7	20	107	58	59	72
12	6	20	19	108	58	72	71
13	7	8	21	109	59	60	73
14	7	21	20	110	59	73	72

15	8	9	22	111	60	61	74
16	8	22	21	112	60	74	73
17	9	10	23	113	61	62	75
18	9	23	22	114	61	75	74
19	10	11	24	115	62	63	76
20	10	24	23	116	62	76	75
21	11	12	25	117	63	64	77
22	11	25	24	118	63	77	76
23	12	13	26	119	64	65	78
24	12	26	25	120	64	78	77
25	14	15	28	121	66	67	80
26	14	28	27	122	66	80	79
27	15	16	29	123	67	68	81
28	15	29	28	124	67	81	80
29	16	17	30	125	68	69	82
30	16	30	29	126	68	82	81
31	17	18	31	127	69	70	83
32	17	31	30	128	69	83	82
33	18	19	32	129	70	71	74
34	18	32	31	130	70	84	83
35	19	20	33	131	71	72	85
36	19	33	32	132	71	85	84
37	20	21	34	133	72	73	86
38	20	34	33	134	72	86	85
39	21	22	35	135	73	74	87
40	21	35	34	136	73	87	86
41	22	23	36	137	74	75	88
42	22	36	35	138	74	88	87
43	23	24	37	139	75	76	89
44	23	37	36	140	75	89	88
45	24	25	38	141	76	77	90
46	24	38	37	142	76	90	89
47	25	26	39	143	77	78	91
48	25	39	38	144	77	91	90

49	27	28	41	145	79	80	93
50	27	41	40	146	79	93	92
51	28	29	42	147	80	81	94
52	28	42	41	148	80	94	93
53	29	30	43	149	81	82	95
54	29	43	42	150	81	95	94
55	30	31	44	151	82	83	96
56	30	44	43	152	82	96	95
57	31	32	45	153	83	84	97
58	31	45	44	154	83	97	96
59	32	33	46	155	84	85	98
60	32	46	45	156	84	98	97
61	33	34	47	157	85	86	99
62	33	47	46	158	85	99	98
63	34	35	48	159	86	87	100
64	34	48	47	160	86	100	99
65	35	36	49	161	87	88	101
66	35	49	48	162	87	101	100
67	36	37	50	163	88	89	102
68	36	50	49	164	88	102	101
69	37	38	51	165	89	90	103
70	37	51	50	166	89	103	102
71	38	39	52	167	90	91	104
72	38	52	51	168	90	104	103
73	40	41	54	169	92	93	106
74	40	54	53	170	92	106	105
75	41	42	55	171	93	104	107
76	41	55	54	172	93	107	106
77	42	43	56	173	94	95	108
78	42	56	55	174	94	108	107
79	43	44	57	175	95	96	109
80	43	57	56	176	95	109	108
81	44	45	58	177	96	97	110
82	44	58	57	178	96	110	109

83	45	46	59	179	97	98	111
84	45	59	58	180	97	111	110
85	46	47	60	181	98	99	112
86	46	60	59	182	98	112	111
87	47	48	61	183	99	100	113
88	47	61	60	184	99	113	112
89	48	49	62	185	100	101	114
90	48	62	61	186	100	114	113
91	49	50	63	187	101	102	115
92	49	63	62	188	101	115	114
93	50	51	64	189	102	103	116
94	50	64	63	190	102	116	115
95	51	52	65	191	103	104	117
96	51	65	64	192	103	117	116

21

13 1 0

26 1 0

39 1 0

52 1 0

65 1 0

78 1 0

91 1 0

104 1 0

105 1 1

106 1 0.9914

107 1 0.9659

108 1 0.9238

109 1 0.866

110	1	0.793
111	1	0.707
112	1	0.608
113	1	0.5
114	1	0.382
115	1	0.258
116	1	0.130
1	0	

b) Two-dimensional groundwater flow considering pumping and recharge

The second problem considered is a two dimensional ground water flow problem considering recharge and pumping in a rectangular aquifer domain. The lines of constant potential (equipotential lines) in a 3000 m $\times$ 1500 m rectangular aquifer shown in fig.3 bounded on the long sides by an impermeable material and on the short sides by a constant head of 200 m is obtained by analysing the problem. A river is passing through the aquifer having an infiltration rate of 0.24 m^3 / day / m^2. Two pumps are located at (1000, 670) and (1900, 900) with pumping at rates of Q_1 = 1200 m^3 / day / m and Q_2 =2400 m^3 / day / m respectively. Take co-efficient of permeability k as 50 m^3 / day / m^2.

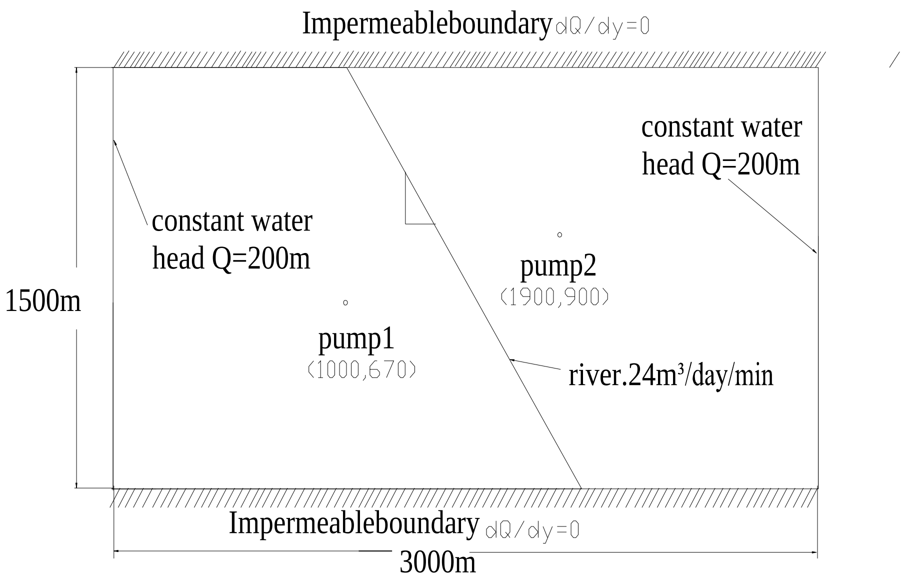

Figure. 3 Geometry and boundary conditions for the groundwater flow problem.

A mesh of 64 triangular elements and 45 nodes is used to model the domain. The river forms the inter-element boundary (33, 35, 37, 39) and (26, 28, 30, 32). In the mesh selected, neither pump is located at a node. If the pumps are located at a node then the rate of pumping is input as the specified secondary variable of the node. When a source is located at a point other than a node, we must calculate its contribution to the nodes. Similarly, the source components due to the distributed line source (i.e., the river) should be computed.

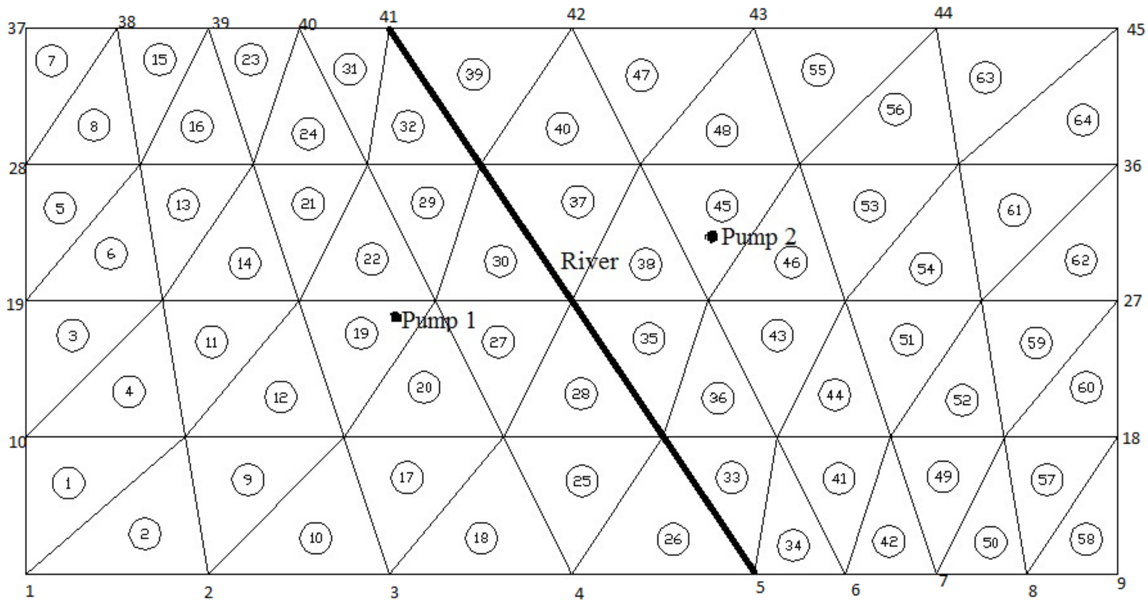

Fig. 4 Finite element mesh with numbering scheme

Program input:

Number of nodes = 45

Number of elements = 64

N0de	X Co-ordinate	Y Co-ordinate	Node	X Co-ordinate	Y Co-ordinate
1	0	0	24	1250	1125
2	0	375	25	1000	1500
3	0	750	26	2250	0
4	0	1125	27	2062	375
5	0	1500	28	1872	750
6	500	0	29	1688	1125
7	375	375	30	1500	1500
8	438	750	31	2500	0

9	313	1125	32	2375	375
10	250	1500	33	2250	750
11	1000	0	34	2125	1125
12	937	375	35	2000	1500
13	875	750	36	2750	0
14	625	1125	37	2687	375
15	500	1500	38	2625	750
16	1500	0	39	2563	1125
17	1312	375	40	2500	1500
18	1125	750	41	3000	0
19	937	1125	42	3000	375
20	750	1500	43	3000	750
21	2000	0	44	3000	1125
22	1750	375	45	3000	1500
23	1500	750			

50

Element	Node1	Node2	Node3	Element	Node1	Node2	Node3
1	1	7	2	33	21	27	22
2	1	6	7	34	21	26	27

3	2	8	3	35	22	28	23
4	2	7	8	36	22	27	28
5	3	9	4	37	23	29	24
6	3	8	9	38	23	28	29
7	4	10	5	39	24	30	25
8	4	9	10	40	24	29	30
9	6	12	7	41	26	32	27
10	6	11	12	42	26	31	32
11	7	13	8	43	27	33	28
12	7	12	13	44	27	32	33
13	8	14	9	45	28	34	29
14	8	13	14	46	28	33	34
15	9	15	10	47	29	35	30
16	9	14	15	48	29	34	35
17	11	17	12	49	31	37	32
18	11	16	17	50	31	36	37
19	12	18	13	51	32	38	33
20	12	17	18	52	32	37	38
21	13	19	14	53	33	39	34
22	13	18	19	54	33	38	39
23	14	20	15	55	34	40	35

24	14	19	20	56	34	39	40
25	16	22	17	57	36	42	37
26	16	21	22	58	36	41	42
27	17	23	18	59	37	43	38
28	17	22	23	60	37	42	43
29	18	24	19	61	38	44	39
30	18	23	24	62	38	43	44
31	19	25	20	63	39	45	40
32	19	24	25	64	39	44	45

10

1 1 200

2 1 200

3 1 200

4 1 200

5 1 200

41 1 200

42 1 200

43 1 200

44 1 200

45 1 200

11	
12	-255.6
13	-229.2
18	-715.2
21	54.08
22	108.17
23	108.17
24	108.17
25	54.08
28	-1440.0
29	-410.4
34	-549.6

3.2 SOFTWARE PACKAGE ANSYS

ANSYS is general-purpose finite element analysis (FEA) software package. ANSYS software allows engineers to construct computer models of structures, machine components or systems; apply operating loads and other design criteria; and study physical responses, such as stress levels, temperature distributions, pressure, etc. The software implements equations that govern the behaviour of elements and solve them; creating a comprehensive explanation of how the system acts as a whole. These results then can be presented in tabulated or graphical form. This type of analysis is typically used for the design and optimization of a system far too complex to analyze by hand. Systems that may fit into this category are too complex due to their geometry, scale, or governing equations. ANSYS also provides a cost-effective way to explore the performance of products or processes in a virtual environment.

3.2.1 Steps to Solve Problems in ANSYS

Like solving any problem analytically, it is essential to define (1) solution domain, (2) the physical model, (3) boundary conditions and (4) the physical properties. Then solve the problem and post processing of the results. The commercial finite element package, ANSYS version 10, was used for modelling and analysis. Below describes the processes in terminology slightly more attune to the software.

- Build Geometry

Construct a two or three dimensional representation of the object to be modeled and tested using the work plane coordinates system within ANSYS.

- Define Material Properties

Now that the part exists, define a library of the necessary materials that compose the object (or project) being modeled. This includes thermal and mechanical properties.

- Selection of element

From the library of ANSYS elements provided suitable element is selected for the analysis of problem.

- Generate Mesh

At this point ANSYS understands the makeup of the part. Now define how the modeled system should be broken down into finite pieces.

- Apply Loads

Once the system is fully constructed, the last task is to burden the system with constraints, such as physical loadings or boundary conditions.

- Obtain Solution

This is actually a step, because ANSYS needs to understand within what state (steady state, transient etc.) the problem must be solved.

- Post-processing of

- the Results

After the solution has been obtained, there are many ways to present ANSYS results, choose from many options such as tables, graphs, and contour plots.

3.2.2 Analysis of problems using the finite element software package ansys

a) Two dimensional heat transfer problem (12x6) solved above is analysed using ANSYS

Modelling and meshing of two dimensional rectangular region was done using element type SOLID 8node77.

b) Stress analysis in a sub-soiler

Modelling and meshing of three dimensional region was done using element type Shell Elastic 4node 63.

CHAPTER IV

RESULTS AND DISCUSSION

A) Heat transfer

A numerical solution for the heat flow equation using finite element method is obtained for an isotropic rectangular region using the program developed in the object-oriented programming language visual c++.The nodal temperature distribution fig.(6) in the domain were obtained and the result obtained were compared with analytical solution and also using the software package ANSYS. We have analyzed a simple heat transfer problem, though the developed program can handle other complex heat transfer problems like heat transfer in heat exchangers, furnaces etc. and also can optimize the design of various heat transfer applications. Three sets of solutions were obtained with triangular element meshes of (3×2), (6×4) and (12×8). By increasing the number of elements, accuracy is found to increase. The nodal temperatures obtained for various meshes are tabulated in table (1), which shows the increase in accuracy with the increase in number of elements. A graph is also plotted for the nodal temperature along lower edge for various meshes and compared with the analytical solution in Fig. (5).

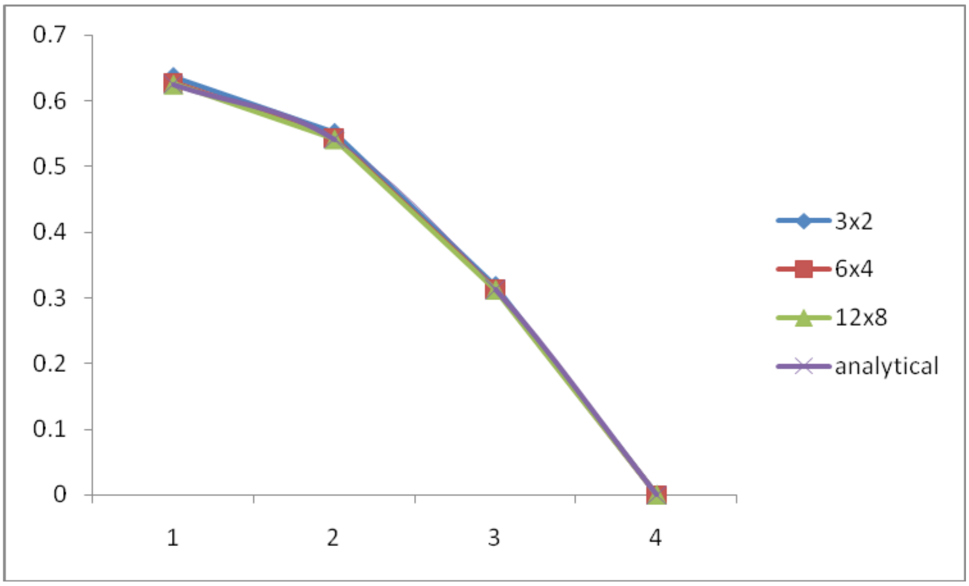

Fig.5 Graphical representation of lower edge node temperature values and analytical solution.

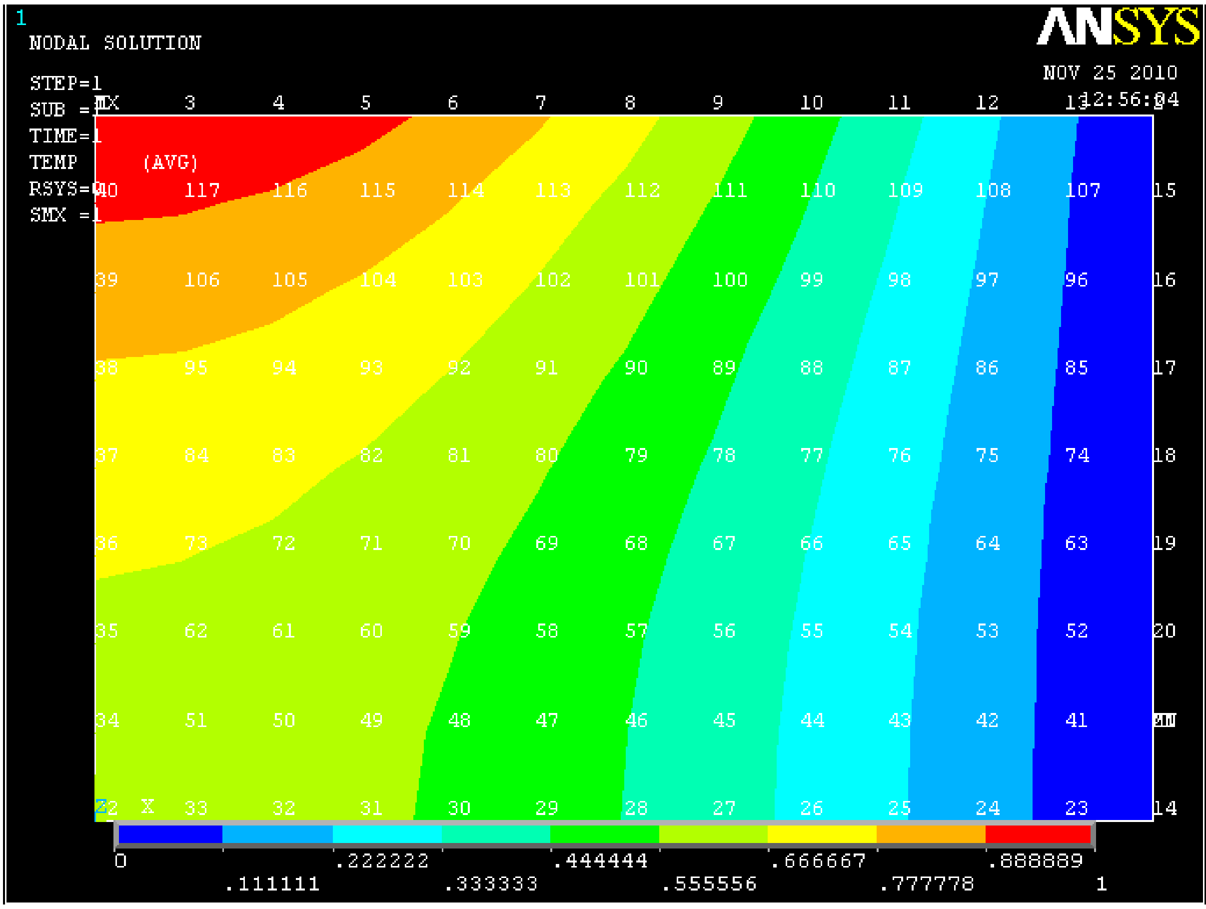

Fig.6 ANSYS output of thermal analysis

Table:1 Comparison of different element meshes with analytical solution and ANSYS

COORDINATES		TRIANGULAR ELEMENT MESH			ANALYTICAL	ANSYS
X	Y	(3X2)	(6X2)	(12X8)		
0.00	0.00	0.6365	0.6275	0.6255	0.6251	0.62549
2.50	0.00	-	0.6061	0.6042	0.6038	0.60417
5.00	0.00	0.5513	0.5435	0.5417	0.5414	0.54166
7.50	0.00	-	0.4437	0.4422	0.4422	0.44224
10.00	0.00	0.3183	0.3138	0.3127	0.3128	0.31269
12.50	0.00	-	0.1624	0.1619	0.1622	0.16185
0.00	2.50	-	0.6489	0.6470	0.6466	0.64699
2.50	2.50	-	0.6268	0.6249	0.6246	0.62494
5.00	2.50	-	0.5620	0.5603	0.5601	0.56028
7.50	2.50	-	0.4589	0.4574	0.4574	0.45744
10.00	2.50	-	0.3245	0.3234	0.3236	0.32343
12.50	2.50	-	0.1679	0.1674	0.1677	0.16741
0.00	5.00	0.7218	0.7145	0.7130	0.7127	0.71298
2.50	5.00	-	0.6901	0.6887	0.6884	0.68868
5.00	5.00	0.6252	0.6188	0.6174	0.6173	0.61741
7.50	5.00	-	0.5053	0.5041	0.5041	0.50406
10.00	5.00	0.3609	0.3573	0.3564	0.3566	0.35637
12.50	5.00	-	0.1849	0.1844	0.1849	0.18444
0.00	7.50	-	0.8289	0.8280	0.8278	0.82801
2.50	7.50	-	0.8005	0.7998	0.7996	0.79978
5.00	7.50	-	0.7179	0.7170	0.7170	0.71700
7.50	7.50	-	0.5861	0.5853	0.5856	0.58532
10.00	7.50	-	0.4144	0.4138	0.4143	0.41378
12.50	7.50	-	0.2143	0.2141	0.2147	0.21405

b)Groundwater

The area under study is discretized with 45 nodes and 64 elements. Bibbin Sunny et al studied the same problem by considering the pumping location to be in the nodes but in this work meshes are selected in such a way that the pumps are not located at the nodes. So the effect of generalized force due to the point source to the neighbouring nodes is also calculated. The effect of river ie. recharge is calculated considering it as a line distributed load. The effect of pumping to the neighbouring nodes is calculated by spatial interpolation. On solution of the problem, the piezometric head, the primary variable, is obtained at various nodes. The greatest drawdown is found to occur at node 28. Based on the maximum drawdown and its location one can easily determine the total volume that may be withdrawn annually from the aquifer and also identify the best potential locations for future well installations and show how the pumping of one or more wells will affect the other wells in the aquifer and surface water bodies.

c)Farm power

Finite element is an effective tool for investigation of 2D analysis in implements. Study focuses on static behaviour of L-shaped Subsoilers. Maximum stress was obtained near the shank's hole as 290MPa. Results showed that fracture probability of the subsoiler near the shank's hole is due to the existence of a bending moment which is produced by the soil resistance force acting on the blades and lower section of the shank. For the subsoiler it is necessary that the body of the subsoiler's shank be strengthened around the holes so that the failure can be avoided. It is also found that upper two shanks holes are not necessary. Hence the material can be reduced

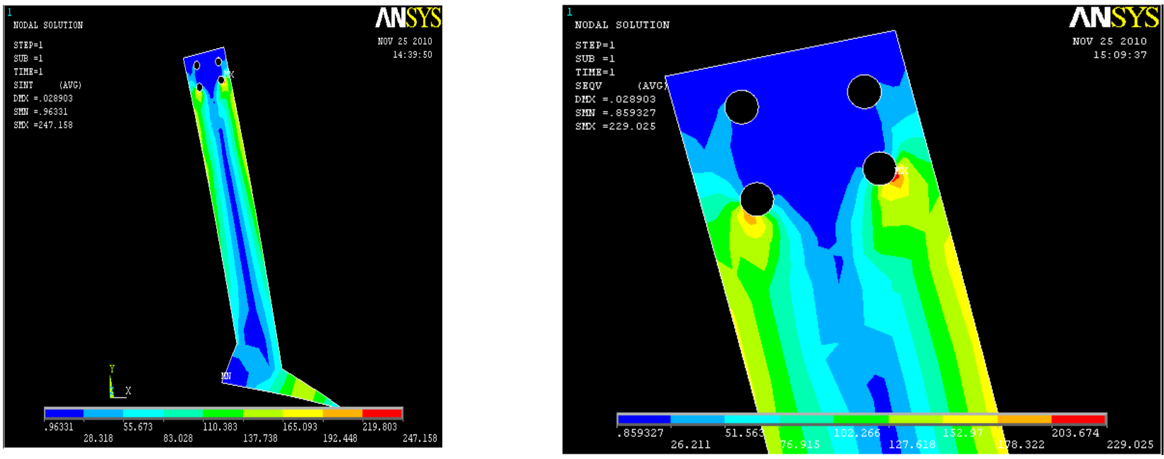

Fig.7 ANSYS output of the subsoiler (Von Mises stress)

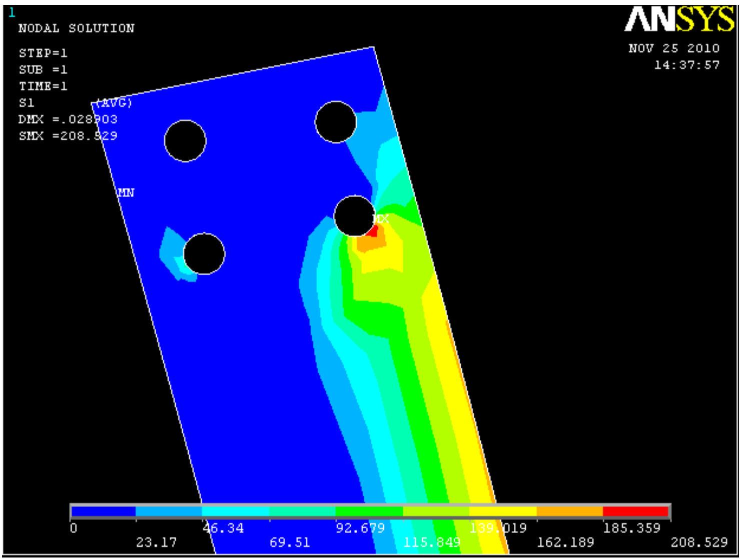

Fig. 8 ANSYS output of subsoiler (Major principle stress)

CHAPTER V

SUMMARY AND CONCLUSION

Agricultural engineering being a field of application of engineering principles in agriculture, deals with many complex problems. The design of simple machine or tool may have many parameters to be considered. The optimal management of groundwater resources is an important task in agricultural engineering. Management of groundwater system means making decisions related to volume of water that can be drawn, location of pumping and artificial recharge wells, if there is groundwater contamination, decision has to be made related to groundwater quality. FEM act as a tool to asses the nature and distribution of subsurface flow. Heat transfer analysis is of great importance and vast application in the field of agricultural engineering especially in the branch of post harvest engineering and food processing. FEM is key in analyzing the cold and hot spots in various food materials of irregular geometries, predicting the microbial distribution and nutrient degradation, optimizing the process and in developing information and nomograms to educate the consumer on heating techniques for food safety and quality.

FEM software helps in simulation of real life situation and practical difficulty in modelling and analysis of a system, and its optimization. FEM has been decisive in decreasing the developing time of products, as the design and development of the prototype became much easier with FEM .

The future scope of our study are Optimization of subsoiler design for various shape and various inclination angles.FEM studies can be done for various implements such as MB plough, cultivators, mowers, machine elements, engine parts etcWe can select suitable shape and materials based on the factor of safety. FEM can be easily utilized for the design of heat exchangers its shape, length, surface area.

A finite element formulation has been developed for two dimensional analysis of Poisson's equation. Various numerical problems in the field of ground water, farm machinery and heat has been analyzed and results compared with available analytical results and using software package ANSYS

CHAPTER VI

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APPENDIX A

// Program for Finite Element Analysis of 2D Ground Water Flow and Heat transfer Problems

```
#include <fstream.h>

#include <iostream.h>

#include <iomanip.h>

#include <math.h>

#include "Mat.h"

#include "Vec.h"

void triangle( void);

dMatrix ecoeff(3,3);

dVector  eQ(3), xl(2);

dVector N(3), Nxi(3);

int  neq;

double delta,Q;

double K, b1, b2, b3, c1, c2, c3;

double X1,X2,X3,Y1,Y2,Y3;

void main(void)

{

        ifstream fin ("2DGW flow.inp");
```

```
        ofstream fout("2DGW flow.out");
    dMatrix gC;
        fout.setf(ios::showpoint);
        fout.setf(ios::floatfield, ios::fixed);
        fout.precision(4);
        int nElems, nNodes, nknownheads;
        int  i, j, k;
        fin >> nNodes >> nElems;
        dVector X(nNodes), Y(nNodes);
        for (i=1; i<=nNodes; ++i)
        {
              fin >> k;
                    fin >> X[k] >> Y[k];
        }
        fout << "\n*********************************\n";
        fout << "\n FINITE ELEMENT ANALYSIS PROGRAM\n";
        fout << "\n*********************************\n\n";
        fout << "Problem Type: 2D Ground Water Flow Problems\n";
        fout << "\nNumber of nodes    =  " << nNodes;
        fout << "\nNumber of elements =  " << nElems;
        fout << "\n\nNodal Coordinates";
```

```
fout << "\n~~~~~~~~~~~~~~~~~~\n";

fout << "\nNode    x-coord.     y-coord.\n";

fout <<"===============================";

for (i=1; i<=nNodes; ++i)

{

        fout << "\n" << setw(4) << i << setw(12);

        fout << X[i] << setw(12) << Y[i] << setw(12);

}

        fin >> K;

iMatrix elemConn(3, nElems);

fout << "\n\nElement Connectivity";

fout << "\n~~~~~~~~~~~~~~~~~~~~~~\n";

fout <<"\n  Elem  Nod1   Nod2   Nod3         \n";

fout <<"===================================\n";

for (i=1; i<=nElems; ++i)

{

        fin >> k;

        for (j=1; j<=3; ++j)

                fin >> elemConn(j, k);

}

for (i=1; i<=nElems; ++i)
```

```
{
    fout << "\n" << setw(4) << i << setw(5);
    for (j=1; j<=3; ++j)
        fout << elemConn(j, i) << setw(7);
}
dVector U(nNodes);
iVector destn( nNodes), destn1(nNodes);
int dof;
fin >> nknownheads;
fout << "\n\nNumber of nodes at which displacement is prescribed  =  " << nknownheads;
for (i=1; i<=nknownheads; ++i)
{
    fin >> k;
        dof = k;
        fin >> destn[k] >> U[dof];
}
neq = 0;
for (j=1; j<=nNodes; ++j)
{
        if (destn[ j] == 0)
        {
```

```
                neq++;
                destn[ j] = neq;
                continue;
            }
            else
                destn[ j] = 0;
    }
    fout << "\n\nThe Destination Array:";
    fout << "\n~~~~~~~~~~~~~~~~~~~~~~~\n";
    fout << "\n  Node         X-dof           \n";
    fout <<"=========================================";
    for (i=1; i<=nNodes; ++i)
    {
        fout << "\n" << setw(4) << i << setw(9);
        fout << destn[ i] << setw(9);
    }
    fout << "\n\nNo. of Degrees of Freedom =  " << neq;
    dVector gLoad(neq);
    dVector gDisp(neq);
    double Q, pres_head[3], eLoad_head[3];
    int nknownQ;
```

```
fin >> nknownQ;
for (i=1; i<=nknownQ; ++i)
{
        fin >> k;
                dof = destn[ k];
                fin >> Q;
                if (dof != 0)
                        gLoad[dof] += Q;
}
int dof1, tot_dof;
dMatrix gcoeff(neq,neq);
for (i=1; i <= nNodes; ++i)
{
        dof = destn[ i];
        double zero = 0.0;
        fout << "\n "<< setw(4) <<i;
                if (dof != 0)
                        fout << setw(15) << gLoad[dof] << setw(15) << dof;
                else
                        fout << setw(15) << zero;
}
```

```
gcoeff = 0;

// Assembly of element matrices

iVector kk(2), kkA(2);

int l,node1,node2, node3, node;

for (int n=1; n<=nElems; ++n)

{

        node1 = elemConn(1, n);

        node2 = elemConn(2, n);

        node3 = elemConn(3, n);

        X1 = X[node1];

X2 = X[node2];

X3 = X[node3];

   Y1 = Y[node1];

   Y2 = Y[node2];

   Y3 = Y[node3];

        b1 = Y[node2]-Y[node3];

        b2 = Y[node3]-Y[node1];

        b3 = Y[node1]-Y[node2];

        c1 = X[node3]-X[node2];

        c2 = X[node1]-X[node3];

        c3 = X[node2]-X[node1];
```

```
delta=  (c3 * b2 - c2 * b3);
triangle();
fout << "\necoeff =" << ecoeff;
dof = 0;
for (i=1; i<=3; ++i)
{
            dof ++;
            int node = elemConn(i, n);
            kk[dof] = destn[ node];
}
tot_dof =3;
dof1 = 0;
for (i=1; i<=3; ++i)
{
      node = elemConn(i,n);
            dof1 ++;
            dof = node ;
            pres_head[dof1] = U[dof];
}
for (i=1; i<=tot_dof; ++i)
{
```

```
}

            eLoad_head[i] = 0.;

            for (j=1; j<=tot_dof; ++j)

            {

                eLoad_head[i] += ecoeff(i,j) * pres_head[j];

            }

        }

    for (int m=1; m<=tot_dof; ++m)

        {

            if (kk[m] <= 0)

                continue;

            k = kk[m];

            gLoad[k] += eQ[m] - eLoad_head[m];

            for (j=1; j<=tot_dof; j++)

            {

                if (kk[j] <= 0)

                    continue;

                  l= kk[j] ;

                gcoeff(k,l) +=  ecoeff(m,j);

            }

        }

}
```

```
            fout << "\ngcoeff ="<< gcoeff;
            fout << "\ngLoad ="<< gLoad;
            gDisp = gcoeff^gLoad;
            fout << "\ngDisp = " << gDisp;
            fout << "\n\nGlobal Displacement Vector";
            fout <<"\n~~~~~~~~~~~~~~~~~~~~~~~~~~~~";
            for (i=1; i<=nNodes; ++i)
            {
                        U[i] = gDisp[destn[i]];
            }
            for (j=1; j<=nNodes; ++j)
            {
                  fout << "\n" << setw(4) << j << setw(15);
                        fout << U[j] << setw(15);
            }
}
void triangle( void)
{
            double dn1dx,dn2dx,dn3dx,dn1dy,dn2dy,dn3dy;
double s11,s12,s13,s21,s22,s23,s31,s32,s33;
            dn1dx = (Y2 - Y3) / delta;
```

```
dn2dx = (Y3 - Y1) / delta;

dn3dx = (Y1 - Y2) / delta;

dn1dy = (X3 - X2) / delta;

dn2dy = (X1 - X3) / delta;

dn3dy = (X2 - X1) / delta;

dMatrix s(3,3);

s11= (dn1dx * dn1dx + dn1dy * dn1dy) * K * delta / 2;

s12= (dn1dx * dn2dx + dn1dy * dn2dy) * K * delta / 2;

s13= (dn1dx * dn3dx + dn1dy * dn3dy) * K * delta / 2;

s22= (dn2dx * dn2dx + dn2dy * dn2dy) * K * delta / 2;

s23= (dn2dx * dn3dx + dn2dy * dn3dy) * K * delta / 2;

s33= (dn3dx * dn3dx + dn3dy * dn3dy) * K * delta / 2;

s21 = s12;

s31 = s13;

s32 = s23;

ecoeff=0;

ecoeff(1,1) = s11;

ecoeff(1,2) = s12;

ecoeff(1,3) = s13;

ecoeff(2,1) = s21;

ecoeff(2,2) = s22;
```

```
        ecoeff(2,3) = s23;

        ecoeff(3,1) = s31;

        ecoeff(3,2) = s32;

        ecoeff(3,3) = s33;

}
```

ABSTRACT

Numerical modelling is becoming an increasingly important tool for analyzing complex problems in the agricultural engineering related problems. Numerical models, particularly the finite difference and finite element methods, were extensively used for modelling such a complex systems. A finite element formulation of two-dimensional groundwater flow and heat transfer problems were developed and a Visual C++ program is used for this purpose to obtain primary unknowns hydraulic head and temperature at the nodes. The domain of the problem was discretized into linear triangular elements. The numerical solutions were obtained for groundwater flow considering pumping and recharge and heat transfer in an isotropic rectangular region. By the analysis of the hydraulic head solution results, one can easily obtain the flow direction vectors, flow velocities and flow rates in different directions and in the same way nodal temperatures can be used to obtain heat flow rates, thermal flux and thermal gradient. ANSYS is a general purpose engineering simulation software package based on the finite element analysis, allowing engineers to refine and validate designs at a stage when cost of making changes is minimal. ANSYS simulation software can predict how product design will operate and manufacturing processes will behave in real world environments.

The analysis in ANSYS consists of three steps the pre-processing, the analysis and the post-processing. By post-processing the obtained results can be presented in a desired form. In this work two problems were analysed using ANSYS, the heat transfer in an isotropic medium and the stress analysis of a sub-soiler. The results obtained by finite element program developed for the heat transfer problem is compared with ANSYS results and also analytical results and results are found to compare well. From the stress analysis of the sub-soiler, it is found that the stress near the upper two holes of the sub-soiler is very small and so the upper two holes can be avoided. FEM improve accuracy, enhance the design and better insight into critical design parameters, virtual prototyping, fewer hardware prototypes, a faster and less expensive design cycle, increased productivity, and increased revenue.