KERALA AGRICULTURAL UNIVERSITY
B.Tech. (Agrl. Engg.) 2022 & Previous Admissions
II Semester Final Examination - September 2023

Sacs.1206   Engineering Mathematics - II (2+1)   **Marks: 50**
                                                                             Time: 2 hours

I   **Choose the correct answer**   **(10x1=10)**

1. Series $1 + r + r^2 + \cdots \infty$ is ....
   - (a) Converges if $|r| < 1$
   - (b) diverges if $|r| < 1$
   - (c) Oscillatory if $|r| < 1$
   - (d) none of these

2. The series $\sum u_n$ of positive terms is convergent or divergent as $\lim\limits_{n \to \infty}\left[n\left(\dfrac{u_n}{u_{n+1}} - 1\right)\right] > 1$ or < 1, then

   this test is known as ....
   - (a) Comparison test
   - (b) Raabe's test
   - (c) D-Alembert's ratio test
   - (d) Cauchy's root test

3. The Fourier series expansion of x in the interval $-1 \le x \le 1$ with periodic condition has ....
   - (a) only sine terms
   - (b) only cosine terms
   - (c) both sine and cosine terms
   - (d) only sine terms and a non-zero constant.

4. The value of a_0 in the cosine series expansion of $f(x) = x$ in $(0,5)$ is ....
   - (a) 2
   - (b) 3
   - (c) 5
   - (d) 10

5. Rational function $f(z) = \dfrac{3 + 2z + z^3}{4 + 5z + 3z^3}$ is ....
   - (a) Analytic
   - (b) Not analytic
   - (c) Analytic only when $3 + 2z + z^3$ does not vanish
   - (d) Analytic only when $4 + 5z + 3z^3$ does not vanish.

6. The Fourier Cosine transform of
   $f(x) = \begin{cases} k, & if\ 0 < x < 1 \\ 0, & if\ x \ge 1 \end{cases}$ is ....
   - (a) $k\sqrt{\dfrac{2}{\pi}}\dfrac{\sin s}{s}$
   - (b) $\sqrt{\dfrac{2}{\pi}}\dfrac{\sin s}{s}$
   - (c) $k\sqrt{\dfrac{2}{\pi}}\dfrac{\cos s}{s}$
   - (d) $\sqrt{\dfrac{2}{\pi}}\dfrac{\cos ks}{s}$

7. The Partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $c \neq 0$ is known as....................
   - (a) One dimensional heat equation
   - (b) One dimensional wave equation
   - (c) Laplace equation
   - (d) None of these

8. The partial differential equation formed from the expression $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ by eliminating arbitrary constants a and b is............
   - (a) $2z = px + qy$
   - (b) $2z = qx + py$
   - (c) $z = px + qy$
   - (d) $z = qx + py$

9. Analytic function with constant modulus is ....
   - (a) Constant
   - (b) Need not be a constant
   - (c) depends on the analytic function
   - (d) None of these.

State True or False

10. " If $f(x)$ is non-periodic and defined in $(-\infty, \infty)$, the Fourier series expansion of $f(x)$ is not possible".

II **Write short notes on ANY FIVE of the following** (5x2=10)

1. Examine the convergence of $\sum_{k=1}^{\infty} \frac{4^k}{k^2}$

2. Find the Fourier sine integral of $f(x) = \begin{cases} 1 & if\ 0 < x < \pi \\ 0 & if\ x > \pi \end{cases}$.

3. Find the half range sine series representation of $f(x) = k$ in $(0, \pi)$.

4. Form the partial differential equation by eliminating the arbitrary constants from $z = xy + y\sqrt{x + a} + b$

5. Write down the possible solutions of one dimensional wave equation.

6. Check whether the function $f(z) = e^x(cosy - isiny)$ is analytic or not.

7. Evaluate $\int_c Re(z)dz$ where C is the shortest path from $1 + i$ to $5 + 5i$

III **Answer ANY FIVE of the following** (5x4=20)

1. Examine the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$ using alternating series test.

2. Test the convergence of $\sum_{k=1}^{\infty} (\frac{2k+1}{4k-3})^k$

3. Find the Fourier series of $f(x) = x$ in $-\pi < x < \pi$.

4. Prove that an analytic function of constant real part is constant.

5. Evaluate $\int_C \frac{1}{z^2+4} dz$ where C is $|z - 2| = 2$

6. What type of singularity have the function $f(z) = e^{1/z}$

7. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$.

IV **Write an essay on ANY ONE of the following** (1x10=10)

1. Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = ly - mx$.

2. Show that $v = (2x - 1)y$ is harmonic and find the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

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