KERALA AGRICULTURAL UNIVERSITY

B.Tech (Food .Engg) 2013 Admission

Ist Semester Final Examination-January 2013

Cat. No: Basc.1102 **Marks: 50**

Title: Engineering Mathematics-I (3+0) **Time: 2hours**

Part I

(Answer all questions)

1. Every square matrix can be uniquely expressed as the sum of a _____ and a _____ matrix.
2. Transpose of the product AB of two matrices A and B is equal to ______.
3. The product of eigen values of a matrix A is equal to

   (a). Trace of A (b). $|A|$ (c). 1 (d). 0
4. The rank of $A=\begin{bmatrix} 1 & 2 & 3 & 4 \\ -2 & 0 & 5 & 7 \end{bmatrix}$ is _____.
5. The nature of the canonical form $2x^2+3y^2+z^2+2xy+4xz-2yz$ is _____.
6. If $u=x^y$, then $\frac{\partial u}{\partial x}$ is

   (a). 0 (b). yx^{y-1} (c). $x^y \log x$ (d). $x^y \log y$
7. The value of $Lt_{x\to\frac{\pi}{2}} \frac{\log \sin x}{\left(\frac{\pi}{2}-x\right)^2}$ is _____.
8. $\Gamma\left(\frac{1}{2}\right)=$_____.
9. Curvature of a straight line is _____.
10. If $J_1=\frac{\partial(u,v)}{\partial(x,y)}$ and $J_2=\frac{\partial(x,y)}{\partial(u,v)}$, then $J_1J_2=$_____.

(10 x 1 = 10)

Part II

(Answer any five questions)

1. Write the matrix associated with the quadratic form $x^2-4y^2+6z^2+2xy-4xz$.
2. Expand e^x in powers of $(x-1)$ up to fourth term.
3. Evaluate $Lt_{x\to 0}\left(\frac{x-\sin x}{x^3}\right)$.
4. Verify Euler's theorem for the function $u=x^3-2x^2y+3xy^2+y^3$.

5. Show that the vectors $(1,2,2)$, $(2,1,-2)$ and $(2,-2,1)$ are linearly independent.

6. Prove that $\int_0^1 x^{m-1}\left(1-x^2\right)^{n-1}dx = \frac{1}{2}\beta\left(\frac{m}{2},n\right)$.

7. Obtain the diagonalised matrix associated with $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

8. If $u = x^2 - 2y$ and $v = x + y$, find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$.

(5 x2=10)

Part III

(Answer <u>any five</u> questions)

1. Find the Eigen values and vectors of the matrix $A=\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

2. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

3. Find the radius of curvature at any point (x,y) on the parabola $y^2 = 4ax$.

4. If $u = x^3 + y^3$, where $x = a\cos t$, $y = b\sin t$; find $\frac{du}{dt}$ and verify by direct substitution.

5. Trace the curve $y^2(2a-x) = x^3$.

6. Evaluate $\int_0^a\int_0^b\int_0^c \left(x^2+y^2+z^2\right) dx\ dy\ dz$.

7. Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 x \cos^4 x\ dx$.

(5 x 4=20)

Part IV

(Answer <u>any one</u> question)

1. Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form and find its nature.

2. Define Beta function and prove that $\beta(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

(1 x 10 = 10)