

KERALA AGRICULTURAL UNIVERSITY
B.Tech (Food. Engg) 2012 Admission
Ist Semester Final Examination- January/February -2013

Cat. No: Basc.1102
Title: Engineering Mathematics -I (3+0)

Marks: 80
Time: 3 hours

PART I

Answer all Questions

10 × 1 = 10

1. If A is a square matrix of order n, then what is determinant of -A.
2. Define symmetric and skew symmetric matrices.
3. If A is a non singular matrix of order n, then what is the rank of A.
4. What is the curvature of a circle of radius r at any point.
5. Evaluate $\lim_{x \rightarrow 1, y \rightarrow 2} \frac{2x^2y}{x^2+y^2+1}$
6. Find $\int_0^{\frac{\pi}{2}} \sin^4 x dx$
7. What is the length of the arc of the curve $y = f(x)$ between the points $x = a$ and $x = b$.
8. What is the relation between beta and gamma functions.
9. What is the value of $\frac{\partial^2 u}{\partial x^2}$ where $u = xy + y^2 + 2xz$.
10. Find $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

PART II

Answer any ten Questions

10 × 3 = 30

1. If $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{pmatrix}$. Find the product AB.
2. Prove that $(AB)^{-1} = B^{-1}A^{-1}$ where A and B are two square matrices of the same order.
3. Test for consistency and then solve $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$.
4. Evaluate $\lim_{x \rightarrow 0} \frac{x e^x - \log(1+x)}{x^2}$
5. Define curvature, radius of curvature, and center of curvature of a curve at any point.
6. If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
7. If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$

8. If $x = r\cos\theta$, and $y = r\sin\theta$, then show that $\frac{\partial(x,y)}{\partial(r,\theta)} = r$.
9. Evaluate $\int_0^{\frac{\pi}{6}} \cos^4(3\theta)\sin^3(6\theta)d\theta$
10. Find the reduction formula for $\int x^n e^{ax} dx$
11. Find the volume of the sphere of radius a.
12. Find the surface area of the solid generated by revolving the cardioid $r=a(1+\cos\theta)$ about the initial line.

PART III

Answer any six Questions

$6 \times 5 = 30$

1. Prove that $A^3 - 4A^2 - 3A + 11I = 0$ where $A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{pmatrix}$
2. Find the inverse of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$
3. Examine the polynomial function given by $f(x) = 10x^6 - 24x^5 + 15x^4 - 40x^3 + 108$ for maximum and minimum values .
4. Expand $\log(1 + \sin^2 x)$ in powers of x as far as the term x^5
5. Find the coordinates of the center of curvature at any point of the parabola $y^2 = 4ax$. Hence show that it's evolute is $27ay^2 = 4(x - 2a)^3$.
6. Trace the curve $r=a \sin 3\theta$
7. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.
8. Evaluate $\int_0^a \frac{x^7 dx}{\sqrt{a^2-x^2}}$

PART IV

Answer any one Question.

$1 \times 10 = 10$

1. a) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$
- b) Prove that if λ is an eigenvalue of A, then $\frac{1}{\lambda}$ is an eigen value of A^{-1}
2. a) By changing the order of integration of $\int_0^\infty \int_0^\infty e^{-(xy)} \sin(px) dx dy$, show that $\int_0^\infty \frac{\sin px}{x} dx = \frac{\pi}{2}$
- b) Evaluate $\int \int r \sin \theta dr d\theta$ over the cardioid $r = \frac{a}{3}(1 - \cos \theta)$ above the initial line .
- c) Define Beta function