KERALA AGRICULTURAL UNIVERSITY

B.Tech (Food.Engg.) 2012 & Previous Admissions

III[rd] Semester Final Examination-February-2017

Cat. No: Base.2108. **Marks:80.00**

Title: Engineering Mathematics- III (2+1) **Time: 3 hours**

Part I Answer all the questions **[10 X 1 = 10]**

1. A vector with zero divergence is called a ------ vector.
2. Define conjugate functions of an analytic function.
3. A point where the function ceases to be analytic is called a ------ point.
4. State true or false. Any solution of the Laplace's equation is called a harmonic function.
5. A function $f(t) = \int_0^\infty A(\omega) \cos \omega t \, d\omega$ is a ------ integral representation of $f(t)$.
6. What is a unit step function?
7. Write Cauchy-Riemann equations.
8. A transformation of the form $w = \frac{az+b}{cz+d}$ is called a ------ transformation.
9. A series of the form $a_0 + a_1(z-a) + a_2(z-a)^2 + \ldots + a_n(z-a)^n + \ldots$ is called a ------ series.
10. A pole of order one is also called a ------- pole.

Part II Answer any ten questions **[10 X 3 = 30]**

1. Given $r = \sin t i + \cos t j + tk$, find $\frac{dr}{dt}$ and $\frac{d^2r}{dt^2}$.
2. If $\phi(x,y,z) = x^2 + y^2 + z^2$, find $\frac{d\phi}{ds}$ in the direction of the vector $4i + 2j - 4k$, at the point $(1,1,2)$.
3. Prove that $div\, r = 3$ and $curl\, r = 0$, where $r = xi + yj + zk$.
4. Find $div\, curl f$ where $f = x^2zi - 2y^3z^2j - xy^2k$.
5. Show that $F\{u(t)e^{-at}\} = \frac{1}{i\omega + a}$ $(a > 0)$.
6. Find the Fourier transform of $u(t)t^k e^{-at}$, where k is a positive integer and $a > 0$.
7. Show that the transformation $u = e^{-2xy} \sin(x^2 - y^2)$ is harmonic.
8. Distinguish between isogonal transformation and conformal transformation.
9. Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4u + 3 = 0$.
10. Use Cauchy's integral formula to evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 2$.
11. Expand $\frac{1}{z^2-3z+2}$ in the region $|z| < 1$
12. Write short notes on singularities and zeros.
13. Evaluate $\oint_C \frac{e^z}{(z+1)^2} dz$ where C is the circle $|z-3| = 3$

Part III Answer any six questions **[6 X 5 = 30]**

1. Show that $f = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational and hence find its scalar potential.

2. Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)dydx$

3. Find the Fourier series expansion of the periodic function $f(x) = x^2, \ -\pi \leq x \leq \pi$ of period 2π.

4. Determine a, b, c, d so that the function $f(z) = (x^2 + axy + by^2) + i(cx^2 + dxy + y^2)$ is analytic.

5. Under the transformation $w = \frac{1}{z}$, find the image of $|z - 2i| = 2$.

6. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the paths a) $y = x$ b) $y = x^2$

7. Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole.

8. Evaluate $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$

Part IV Answer any one question **[1 X 10 = 10]**

1. Verify Greens theorem in the plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$.

2. Evaluate $\oint_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle

   i. $|z| = 1$
   ii. $|z + 1 - i| = 2$
   iii. $|z + 1 + i| = 2$

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