KERALA AGRICULTURAL UNIVERSITY
B. Tech. (Agrl. Engg.) 2024 Admission
III Semester Final Examination – January 2026

BES 2106 **Engineering Mathematics I (3+0)**

Marks: 50
Time: 2 hours

I **Answer the following** **(10x1=10)**

1. Define degree of a differential equation.
2. When a first order first degree differential equation
$$M(x,y)\,dx + N(x,y)\,dy = 0$$
is said to be exact?
3. Write the eigenvalues of the matrix $\begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$.
4. Find the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$, if $f = \frac{x^2+y^2}{\sqrt{x}-\sqrt{y}}$.
5. State the complementary function of the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$.
6. For the first order PDE $yu_x - x\,u_y = 0$, write the equation of the characteristic curves.

Fill in the blanks

7. Rank of an 3×3 invertible matrix is …..
8. The value of $B\left(\frac{1}{2},\frac{3}{2}\right)$ is ….

State True or False:

9. "If a 2×2 matrix having distinct eigenvalues, then the matrix is diagonalizable.''
10. "$T: \mathbb{R} \rightarrow: \mathbb{R}$, be a function defined by $T(x) = 2x + 1$ is a linear transformation.''

II **Write short notes on ANY FIVE of the following** **(5x2=10)**

1. Find the integrating factor of the differential equation
$$\frac{dy}{dx} + \tan x\; y = e^x$$
2. Evaluate $\iint_R (x+y)dA$, where R is the rectangle $0 \le x \le 2, 0 \le y \le 1$.
3. Evaluate the integral by changing the order of integration:
$$\int_0^1 \int_y^1 e^{x^2}\,dx\,dy.$$
4. Let A be an 3×3 matrix whose eigenvalues are $-1, 0, 1$. Then find the eigenvalues of the matrix A^3.
5. Let A be an $m \times n$ matrix with real entries. Then write the condition in terms of rank, when the system $AX = b$ has unique solution, for $b \in R^m$.
6. Find the local minima of the function $f(x,y) = x^2 + y^2 - 4x - 6y + 13$.
7. Write the Maclaurin's series expansion of $\sin x$ about $x = 0$.

III **Answer ANY FIVE of the following** **(5x4=20)**

1. Solve the Clairaut's differential equation: $y = px + p^2$.
2. Solve the system of equation using Gauss Elimination method:
$$2x + y - z = 1;\ -3x - y + 2z = -4\ ;\ x + 2y + z = 5$$

3. Find the rank of the matrix: $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 0 & 1 & 2 & 3 \\ 1 & 3 & 5 & 7 \end{pmatrix}$

4. Solve the differential equation using variation of parameter
$\frac{d^2 y}{dx^2} + 4y = \tan 2x$.

5. If $u = f(r)$ and $x = r\cos\theta, y = r\sin\theta$, prove that
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

6. Evaluate: $\iiint_R \frac{dxdydz}{(x+y+z+1)^3}$, where R is the region bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

7. Solve the PDE using Charpit's method:
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2$$

IV **Write an essay on ANY ONE of the following** **(1x10=10)**

1. Diagonalize the matrix : $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. Hence find an invertible matrix P such that $P^{-1}AP = D$ for some diagonal matrix D.

2. Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$, where $P_n(x)$ is the solution of the Legendre differential equation.

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